Chapter 3 – Quadratic Functions

3.2 – The Maximum or Minimum of Quadratic Functions

One very important aspect of quadratic functions is that they all have either a maximum (if the associated parabola opens down) or a minimum (if the parabola opens up). Max/Min's have so many applications in the real world that it's **ridiculous**.

The **BIG QUESTION** we are faced with is this:

How do we find the Maximum or Minimum Value for some given Quadratic?

Example 3.2.1

To find a minimum (or maximum) of a quadratic, **you are NOT allowed to**

point at it and say "There it is"

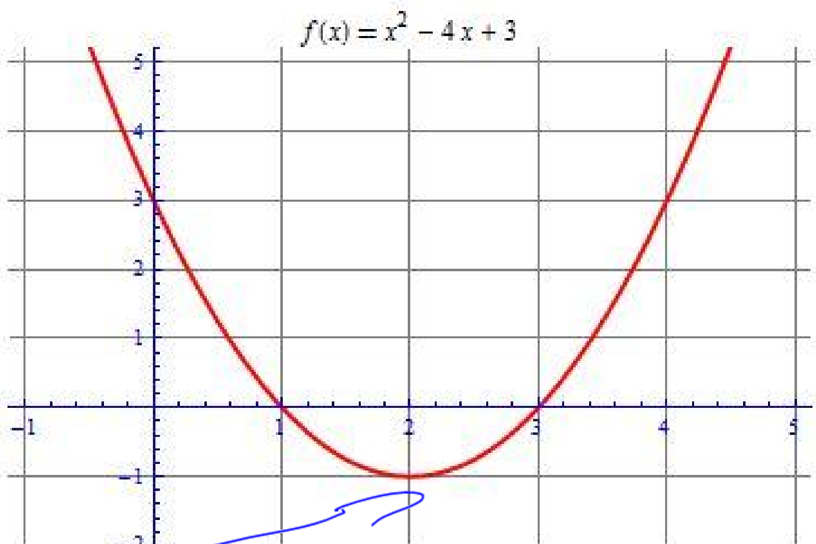


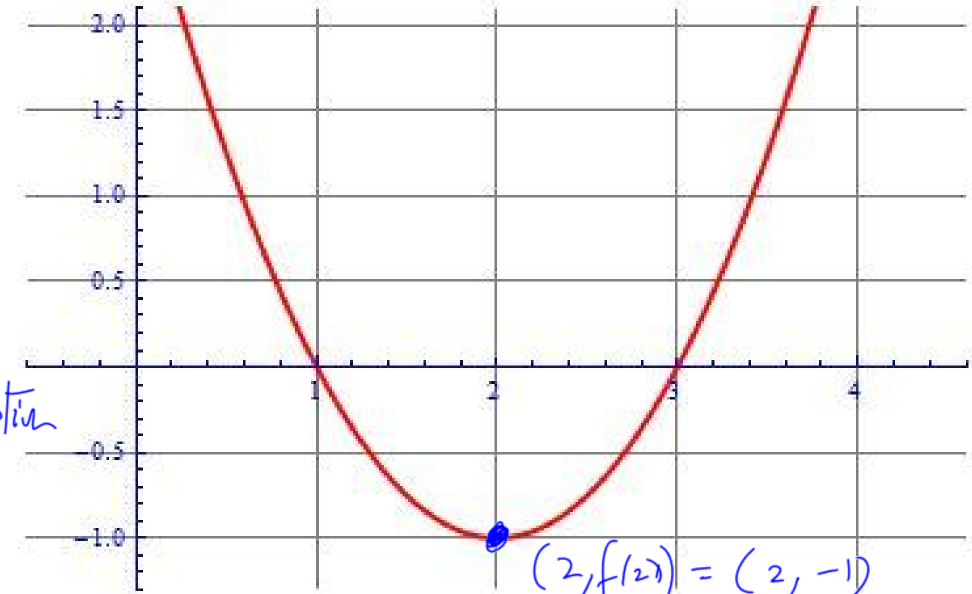
Figure 3.2.1

And, anyway

The vertex is NOT the max/min
The vertex contains two bits of information

$(x, f(x))$

domain is WHERE
RANGE is WHAT



$(2, f(2)) = (2, -1)$

Figure 3.2.1b

We write: $f(x)$ has a min of -1 at $x = 2$.

So, we do need to find the vertex, but we also need to **keep in mind what the numbers associated with the vertex mean**.

In order to find the vertex using algebra, we will consider three techniques:

1) **Using the Zeros**, to find the axis of symmetry, and then the vertex (this is the easiest technique, **assuming we can factor the quadratic**).
2) **Completing the square** to find the vertex (this is the toughest technique, but it's nice because you **end up with the quadratic in vertex form**).
3) Use **Partial Factoring** to find the axis of symmetry, and then the vertex.

Note: We can also use graphing calculators to find the max/min of a quadratic!

Example 3.2.2

for $f(x)$ we will have a max since "a" $= -3 < 0$

Determine the max or min value for the function $f(x) = -3x^2 - 12x + 15$ by finding **THE ZEROS** of the quadratic.

Techniques
factor
Quadratic formula
Graphing Tech

$f(x) = -3(x^2 + 4x - 5)$
$= -3(x+5)(x-1)$

$\therefore$ The zeros are $x = -5, +1$

The AoS: $x = \frac{-5+1}{2}$
$= -2$

$\therefore$ The vertex has coordinates $(-2, f(-2))$

we need to calculate this number

$f(-2) = -3(-2)^2 - 12(-2) + 15$
$= 27$

$\therefore$ The vertex is $(-2, 27)$

$f(x)$ has a max of 27 at $x = -2$.

Example 3.2.3

COMPLETE THE SQUARE to find the vertex of the quadratic and state where the max (min) is and what the max (min) is.

$$g(x) = 2x^2 + 8x - 5$$

Algorithm

① Get "x^2" isolated by factoring "a" but leave the constant alone

② Look at the number on the "x" term
- divide it by 2
- square that – then add it in, subtract it off

$g(x) = 2(x^2 + 4x) - 5$ $\quad \div 2 = 2 \Rightarrow (2)^2 = 4$

$\Rightarrow g(x) = 2(\underbrace{x^2 + 4x + 4}_{\text{perfect square}} - 4) - 5$ $\quad$ zero my hero!

$\Rightarrow g(x) = 2((x+2)^2 - 4) - 5$

$\Rightarrow g(x) = 2(x+2)^2 - 8 - 5$

$\Rightarrow g(x) = 2(x+2)^2 - 13$ $\quad$ vertex form!

$\therefore$ The min is -13 and is at $x = -2$

Example 3.2.4

Using **PARTIAL FACTORING** determine the axis of symmetry. Then find the vertex and state the min or max value.

$$h(x) = 5x^2 + 15x - 3$$

Algorithm

① Common factor the 1st two terms (leave the constant alone)

② Find the zeros of the "partially factored" part

③ Use those domain values to find the AoS

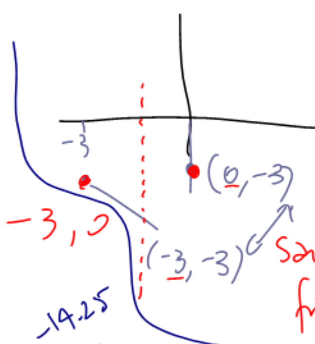
$h(x) = 5x(x+3) - 3$

zeros of $5x(x+3)$ are $x = -3, 0$

$\therefore$ AoS: $x = \frac{-3+0}{2} = -\frac{3}{2}$

$\therefore$ Vertex is $\left(-\frac{3}{2}, h\left(-\frac{3}{2}\right)\right) = \left(-\frac{3}{2}, -\frac{57}{4}\right)$ $\quad -14.25$

$\therefore h(x)$ has a min of $-\frac{57}{4}$ at $x = -\frac{3}{2}$.



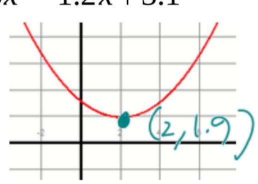
Same find value

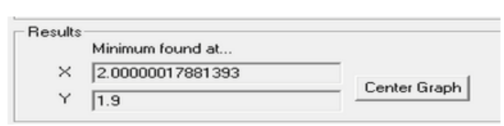
$\Rightarrow$ AoS is between the x-values

Example 3.2.5

Using graphing technology, determine the max/min value of the quadratic

$$f(x) = 0.3x^2 - 1.2x + 3.1$$





$\therefore f(x)$ has a min of 1.9 at $x = 2$.

Class/Homework

Pg. 153 #1, 3, 4abc, 6, 7bc, 8, 9, 11 (ask for help on c if you feel the need!)