Chapter 3 – Quadratic Functions

3.6 – Zeroes of Quadratic Functions

Before we begin, let's think about a couple of things…

Remember – Functions can be described as a set of **ordered pairs**, where the "ordered pair" is a pair of numbers: a **domain value** and a **range value** which can look like $(x, f(x))$. We have talked about the vertex of a parabola. Consider a parabola opening down (which means it will have a maximum value.

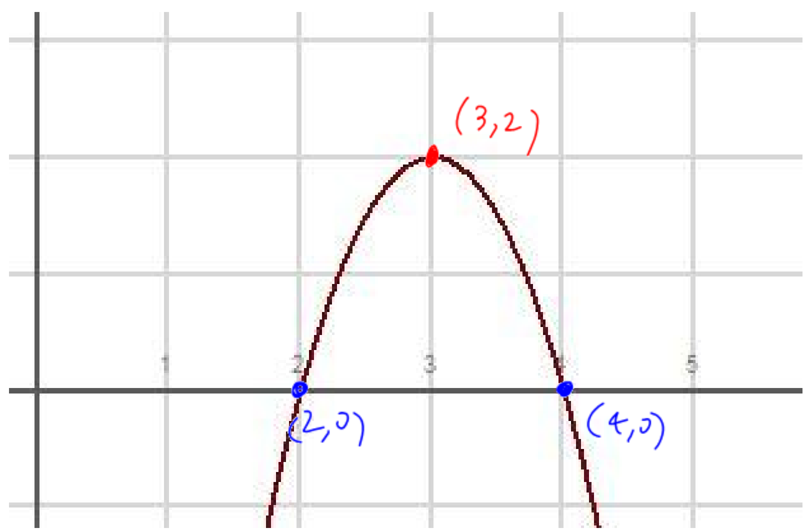
The vertex of that parabola is NOT the maximum. Instead, the vertex is a POINT which is made up of two special numbers. The domain value is WHERE the max occurs and the functional value (the "y" value) is the maximum.

When we talk about the ZEROS of a quadratic we need to understand what we mean by that. Consider the sketch of the graph of the quadratic function $f(x) = -2(x-3)^2 + 2$

A zero has two meanings

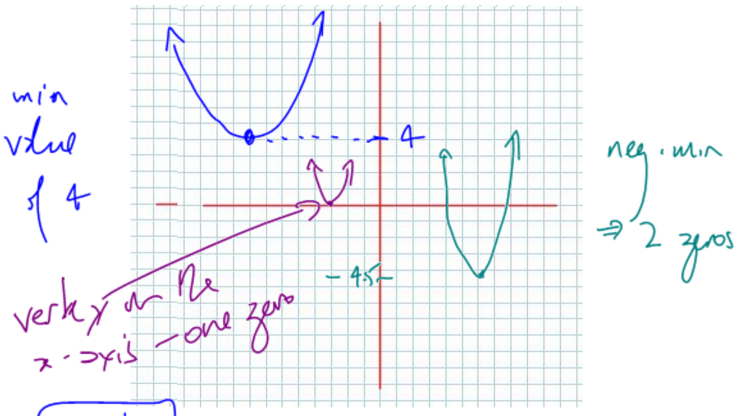
① The point $(x, 0)$

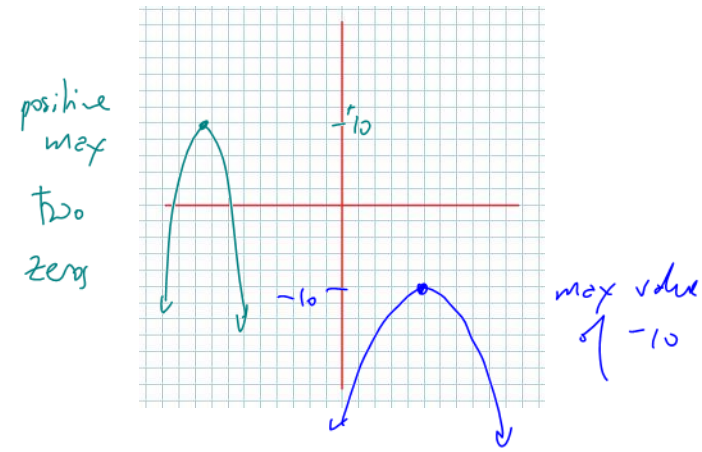
② The domain value 'x', where $f(x) = 0$



A zero is a point where the fnl value is zero.

Q. Do all quadratics have 2 zeros? NO!!!!!!!





positive min ⇒ no zeros!

negative max ⇒ no zeros

Finding the Zeros of a Quadratic

We can find those pesky zeros in a number of ways:

1) Writing the quadratic in zeros form (FACTORED FORM is "easier")
2) Writing the quadratic in vertex form, and doing some algebra
3) Using the quadratic formula (but the quadratic MUST BE IN STANDARD FORM - $f(x) = ax^2 + bx + c$)
4) Using graphing technology (LAME)

Complete the square

Example 3.6.1

Determine the zeros:

a) $f(x) = x^2 - 3x - 4$

$f(x) = (x-4)(x+1)$

$\therefore$ The zeros are: $x = 4$, $x = -1$

b) $g(x) = 2x^2 + x - 1$

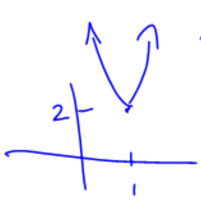
$g(x) = (2x-1)(x+1)$

$\therefore$ The zeros are: $x = \frac{1}{2}$, $x = -1$

Example 3.6.2

Determine why the quadratic $f(x) = 2(x-1)^2 + 2$ has no zeros.

$f(x)$ has a positive minimum (min is +2 at $x = 1$)



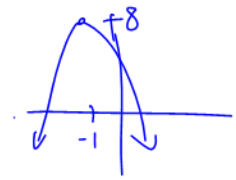
Example 3.6.3

Determine the zeros of $g(x) = -(x+1)^2 + 8$

Note that g(x) has a +ve max (of 8 at x = -1)

To find the zeros from vertex form

- set $g(x) = 0$
- solve for x



$0 = -(x+1)^2 + 8$

$\Rightarrow (x+1)^2 = 8$

$x+1 = \pm\sqrt{8}$

$\therefore x+1 = \pm 2\sqrt{2}$

$\therefore x = +2\sqrt{2} - 1$ or $x = -2\sqrt{2} - 1$

write out both zeros.

Example 3.6.4

Using the quadratic formula, determine the zeros of the quadratic:

In case you've forgotten, the quadratic formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

a) $f(x) = 2x^2 + 3x - 7$

$a = 2 \quad b = 3 \quad c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{3^2 - 4(2)(-7)}}{2(2)}$$

$$= \frac{-3 \pm \sqrt{9 + 56}}{4}$$

$$= \frac{-3 \pm \sqrt{65}}{4}$$

$\therefore$ the zeros are

$x = \dfrac{-3 + \sqrt{65}}{4}$ or $x = \dfrac{-3 - \sqrt{65}}{4}$

b) $g(x) = 3x^2 - 2x + 4$

$a = 3 \quad b = -2 \quad c = 4$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(4)}}{2(3)}$$

$$= \frac{2 \pm \sqrt{-44}}{6}$$

$\therefore$ no zeros

The Discriminant

The Discriminant of the quadratic formula is called the discriminant because it discriminates between the number of zeros a quadratic may have (0, 1, 2 zeros)

The Discriminant is $b^2 - 4ac$

1) If $b^2 - 4ac > 0$ we have 2 (real) zeros

2) If $b^2 - 4ac = 0$ we have 1 (real) zero : $x = \frac{-b}{2a}$

3) If $b^2 - 4ac < 0$ we have 0 zeros

Example 3.6.5

Determine the number of zeros using the discriminant:

a) $f(x) = 2x^2 + 3x - 2$

$a=2 \quad b=3 \quad c=-2$

$b^2 - 4ac = 3^2 - 4(2)(-2)$

$= 9 + 16$

$= 25 > 0 \Rightarrow 2$ zeros

b) $g(x) = -x^2 + 4x - 4$

$a=-1 \quad b=4 \quad c=-4$

$b^2 - 4ac = 4^2 - 4(-1)(-4)$

$= 16 - 16$

$= 0$

$\therefore$ 1 zero

c) $h(x) = 3x^2 + 5x + 6$

$b^2 - 4ac = (5)^2 - 4(3)(6)$

$= 25 - 72 < 0 \quad \therefore$ 0 zeros.

Class/Homework – Pg 185 – 186 #1 – 3abc, 6 – 9 (these are a bit tricky…ask for help!)