Chapter 3 – Quadratic Functions

3.8 – Linear-Quadratic Systems

Recall from Grade 10 that solving a **S**ystem **o**f **L**inear **E**quations could be interpreted to mean finding the point of intersection of the two lines. The solution to a **SoLE** is a point, (x, y). From an algebraic point of view, we have two techniques for solving a SoLE:

1) Substitution
2) Elimination

Example 3.8.1

Solve the SoLE

$$2x+3y=7 \quad (1)$$
$$x-2y=-7 \quad (2)$$

Graphically

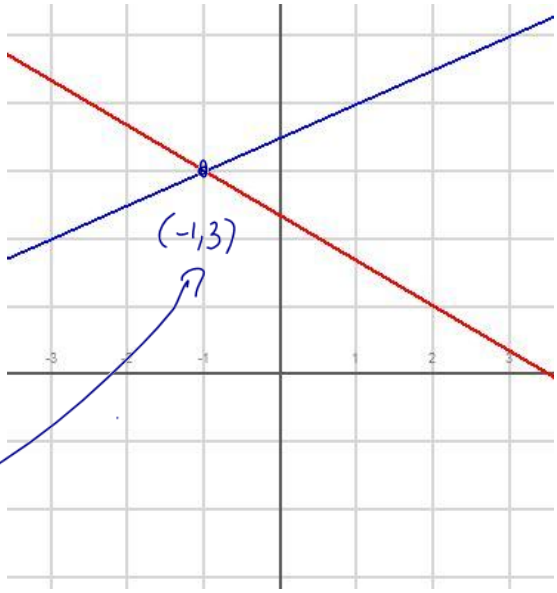


Figure 3.8.1

Algebraically

(2) × 2 $\quad 2x - 4y = -14$ (3)

$2x + 3y = 7$ (1)

(3) − (1) $\quad -7y = -21$

$\Rightarrow y = 3$

Sub $y = 3$ into (2)

$x - 2(3) = -7$

$x - 6 = -7$

$x = -1$

$\therefore$ our soln is $(-1, 3)$

Solving a Linear-Quadratic System is more difficult, but we have the tools to succeed!

We will need to make use of (at least) one Property (or Rule) of Algebra:

The Transitive Property of Equality

Rule: Given three numbers (or more generally, three mathematical objects) a, b, and c, and **if** $c = a$ and $c = b$, **then** $a = b$.

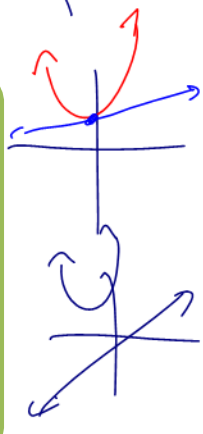
Example: If $f(x) = -2x - 4$, and $f(x) = x^2 - 3x - 10$, then $x^2 - 3x - 10 = -2x - 4$

Example 3.8.2

Solve the Linear-Quadratic System given directly above.

Note: **Solving a Linear-Quadratic System** will amount **to finding the solution(s) to a quadratic equation**. For L-QS`s we can therefore have 0, 1, or 2 solutions.

We will apply the techniques for solving quadratic equations!



$x^2 - 3x - 10 = -2x - 4$

"stuff = 0"

$x^2 - x - 6 = 0$

$\Rightarrow (x-3)(x+2) = 0$

$\therefore x = 3$ or $x = -2$

we need the x's — solns to L-Q syst. are POINTS

soln is $(3, -10)$, $(-2, 0)$ use the linear fn to get the "y-values"

Example 3.8.3 (#2c, on Page 198 from your text)

Determine the point(s) of intersection of the two functions algebraically:

$f(x) = 3x^2 - 2x - 1,\ g(x) = -x - 6$

points of intersection $\Rightarrow f(x) = g(x)$

$\Rightarrow 3x^2 - 2x - 1 = -x - 6$

$\Rightarrow 3x^2 - x + 5 = 0$ diff $\Rightarrow$ QF.

$x = \dfrac{+1 \pm \sqrt{(-1)^2 - 4(3)(5)}}{2(3)} = \dfrac{1 \pm \sqrt{-59}}{6}$

$\therefore$ no solns

Example 3.8.4

Determine the number of points of intersection without solving the System:

$f(x) = x^2 + 2x + 14,\ g(x) = 8x + 5$ (Hint: To solve this problem you must be very discriminating)

discriminant $ax^2 + bx + c = 0$

$\Rightarrow x^2 + 2x + 14 = 8x + 5$

$\Rightarrow x^2 - 6x + 9 = 0$

$a = 1 \quad b = -6 \quad c = 9$

$b^2 - 4ac$

$= (-6)^2 - 4(1)(9)$

$= 36 - 36 = 0$

$\therefore$ There is one point of intersection

Example 3.8.5 (#9 on Page 199 in your text)

9. Determine the value(s) of k such that the linear function $g(x) = 4x + k$ does not intersect the parabola $f(x) = -3x^2 - x + 4$.

$\Rightarrow$ 0 solns $\Rightarrow$ discriminant < 0

$-3x^2 - x + 4 = 4x + k$

$\Rightarrow -3x^2 - 5x + (4 - k) = 0$

$a = -3 \quad b = -5 \quad c = 4 - k$

$b^2 - 4ac < 0$

$(-5)^2 - 4(-3)(4 - k) < 0$

$25 + 12(4 - k) < 0$

$25 + 48 - 12k < 0$

$73 < 12k$

$\frac{73}{12} < k$

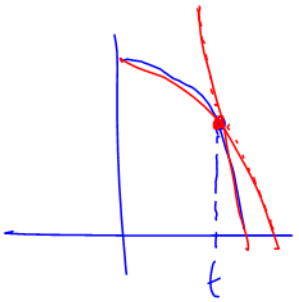
$\therefore k > \frac{73}{12}$

Example 3.8.6 (#10 in your text)

10. A daredevil jumps off the CN Tower and falls freely for several seconds before releasing his parachute. His height, $h(t)$, in metres, t seconds after jumping can be modelled by

$h_1(t) = -4.9t^2 + t + 360$ before he released his parachute; and
$h_2(t) = -4t + 142$ after he released his parachute.

How long after jumping did the daredevil release his parachute?



$-4.9t^2 + t + 360 = -4t + 142$

$\Rightarrow -4.9t^2 + 5t + 218 = 0$ inadmissable

by Desmos: $t = -6.179$ or $t = 7.2$

$\therefore$ after 7.2 seconds the parachute is opened.

Class/Homework: Pg198 – 199 #1ab, 2ab, 3, 4bcd, 6, 8, 11 (tangent means touching at one point!), 12