Chapter 4 – Exponential Functions

4.2 – Integer Exponents

Before beginning, we should quickly review (***ominous music plays***):

THE POWER LAWS

Consider a typical "power" a^n. We call "***a***" the BASE. We call "***n***" the EXPONENT and the entire expression a^n is called a POWER

The Laws: Given the powers a^m and a^n, with exponents *m* and *n*, and the number $\frac{a}{b}$, then $b \neq 0$

same base

1) $1^m = 1$

2) $a^1 = a$

3) $a^m \cdot a^n = a^{m+n}$

4) $a^m \div a^n = a^{m-n}$

5) $a^0 = 1$

6) $(a^m)^n = a^{m \cdot n}$

7) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

8) $(ab)^m = a^m \cdot b^m$

eg $(2x^3)^4 = 2^4(x^3)^4$
$= 16x^{12}$

Until now, for the most part, the exponents you've been working with have always been **NATURAL NUMBERS**. But, we now will examine **INTEGER EXPONENTS**!!

Additional Power Laws:

recall $(-2)^3 = (-2)(-2)(-2) = -8$

9) $a^{-n} = \frac{1}{a^n}$ = NEGATIVE EXPONENTS RECIPROCATE

$= \left(\frac{1}{a}\right)^n = \underbrace{\left(\frac{1}{a}\right)\left(\frac{1}{a}\right)\left(\frac{1}{a}\right)\cdots\left(\frac{1}{a}\right)}_{n \text{ factors}}$

10) $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m = \frac{b^m}{a^m}$

eg $\frac{3x^2y^{-3}}{z^{-4}} = \frac{3x^2z^4}{y^3}$

11) $\frac{a^{-m}}{b^{-n}} = \frac{\frac{1}{a^m}}{\frac{1}{b^n}} = \frac{1}{a^m} \times \frac{b^n}{1} = \frac{b^n}{a^m}$

Example 4.2.1

Write each expression as a single power with a positive exponent:

a) $(4)^{-5} = \left(\frac{1}{4}\right)^5 = \frac{1}{4^5} = \frac{1}{1024}$

b) $\left(\frac{3}{2}\right)^{-4} = \left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4}$

c) $\frac{\cancel{7^3}\,1}{\cancel{7^9}\,6} = 7^{-6} = \frac{1}{7^6}$ $= \frac{1}{7^6}$

Example 4.2.2

Simplify, then evaluate each expression and state your answers in rational form:

a) $3^5(3^{-2})$ $= 3^3$ $= 27$

b) $\left(2^{-3}\left(2^4\right)\right)^{-5}$ $a^m \cdot a^n$ 2^{-3+4} $= 2^1$ $= \left(2^1\right)^{-5}$ $= \frac{1}{2^5}$ $= \frac{1}{32}$

$= 2^{15} \cdot 2^{-20}$ $= 2^{-5}$ $= \frac{1}{2^5}$ $= \frac{1}{32}$

c) $\frac{5^{-3}}{\left(5^2\right)^{-2}}$ $(a^m)^n$ $= \frac{5^{-3}}{5^{-4}}$ $= 5^{-3-(-4)}$ $= 5^1$ $= \frac{5^4}{5^3}$ $= 5^{4-3}$ $= 5^1 = 5$

Example 4.2.3

Evaluate and express in $\boxed{\text{rational form:}}$ ⇒ "$\frac{a}{b}$"

a) $3^2\left(6^{-3}\right) = \frac{3^2}{1}\cdot\frac{1}{6^3}$

$= \frac{3^2}{6^3}$

$= \frac{9}{216}$

$= \frac{1}{24}$

$\frac{3^2}{(2\cdot 3)^3}$

$= \frac{\cancel{3^2}}{2^3\cdot \cancel{3^3}\,1}$

$= \frac{1}{24}$

b) $2^{-3}+10^{-3}-3\left(5^{-3}\right)$ ⇒ $\frac{3}{1}\cdot\frac{1}{5^3}$

$= \frac{1}{2^3} + \frac{1}{10^3} - \frac{3}{5^3}$

$= \frac{1}{8} + \frac{1}{1000} - \frac{3}{125}$

$= \frac{125}{1000} + \frac{1}{1000} - \frac{24}{1000}$

$= \frac{102}{1000} = \frac{51}{500}$

$\frac{1}{2^3} + \frac{1}{2^3\cdot 5^3} - \frac{3}{5^3}$

$\frac{5^3 + 1 - 3\cdot 2^3}{2^3\cdot 5^3}$

$= \frac{125+1-24}{1000}$

etc

c) $13^{-5}\times\left(\frac{13^2}{13^8}\right)^{-1} = \frac{1}{13^5}\cdot\left(\frac{13^8}{13^2}\right)$

$= \frac{13^8}{(13^5)(13^2)}$

$= \frac{13^8}{13^7}$

$= 13$

Example 4.2.4

Evaluate using the laws of exponents (the power rules):

a) $3^2\times 9^{-3}\div 3^{-7}$

$\frac{3^2}{1}\times\frac{1}{9^3}$

$= \frac{3^2}{9^3} \div \frac{1}{3^7}$

$= \frac{3^2}{9^3}\times\frac{3^7}{1}$

$= \frac{3^9}{9^3}$

$= \frac{19683}{729} = 27$

$3^2\times(3^2)^{-3}\div 3^{-7}$

$= 3^2\times 3^{-6}\div 3^{-7}$

$= 3^{2+(-6)-(-7)}$

$= 3^3$

$= 27$

b) $\dfrac{4^{-2}+3^{-1}}{5^{-1}+2^{-2}} = \dfrac{\frac{1}{16}+\frac{1}{3}}{\frac{1}{5}+\frac{1}{4}} = \dfrac{\frac{1(3)}{48}+\frac{1(16)}{48}}{\frac{1(4)}{20}+\frac{1(5)}{20}}$

$= \dfrac{\frac{3+16}{48}}{\frac{4+5}{20}}$

$= \dfrac{\frac{19}{48}}{\frac{9}{20}} = \frac{19}{\cancel{48}\,12}\times\frac{\cancel{20}\,5}{9}$

$= \frac{95}{108}$

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