

Homework Check

pg 222

8. Evaluate. Express answers in rational form.

a) $5^2(-10)^{-4}$

c) $\frac{12^{-1}}{(-4)^{-1}}$

e) $(8^{-1})\left(\frac{2^{-3}}{4^{-1}}\right)$

b) $16^{-1}(2^5)$

d) $\frac{(-9)^{-2}}{(3^{-1})^2}$

f) $\frac{(-5)^3(-25)^{-1}}{(-5)^{-2}}$

$$\frac{(-5)^3(-25)^{-1}}{(-5)^{-2}}$$

$$= \frac{-125 \left(\frac{1}{-25}\right)}{\left(\frac{1}{-5}\right)^2}$$

$$= \frac{5}{\frac{1}{25}}$$

$$= \frac{5}{1} \times \frac{25}{1} = 125$$

$$\left(\frac{1}{-5}\right)^2 = \frac{1^2}{(-5)^2}$$

$$\frac{(-5)^3(-5)^{-2}}{(-5)^{-2}}$$

$$= \frac{(-5)^3(-5^2)^{-1}}{(-5)^{-2}}$$

$$= \frac{(-5)^5}{(-5)^2}$$

$$= \frac{(-1)(5)^5}{-5^2} = -\frac{(-1)^5(5)^5}{5^2}$$

$$= -\frac{(-1)(5^3)}{1} = +125$$

7 b) $(-3)^{-1} + 4^0 - 6^{-1}$

$$= -\frac{1}{3} + 1 - \frac{1}{6}$$

$$= \frac{-2 + 6 - 1}{6} = \frac{3}{6} = \frac{1}{2}$$

$$-\frac{1}{2} = \frac{1}{-2} = -\frac{1}{2}$$

Chapter 4 – Exponential Functions

$$2^3 = 2 \times 2 \times 2$$

4.3 – Rational Exponents

$$2^{-3} = \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

A **rational** exponent can be a **fraction**. For example we can consider the number $(16)^{\frac{3}{4}}$. Of course, the question we need to ask is:

What the rip is that thing??

As you know, a fraction has two parts: a numerator, and a denominator. When a fraction is used as an exponent, the two parts of the fraction carry two related (but different) meanings in terms of “powers”.

Recall that 4^3 **means** $4 \times 4 \times 4$. Now $4^{\frac{1}{2}}$ **does not mean** $4 \div 4$! Your text has a nice explanation of the meaning of numbers like $4^{\frac{1}{2}}$. See (i.e. **READ** examples 1 and 2 on pages 224 and 225. For now, we will simply take the meaning of

roots: $4^{\frac{1}{2}}$ means - some # "a" which when multiplied by itself twice, the answer would be 4

Definition 4.3.1

Given a power with a “rational” (fractional) exponent $a^{\frac{m}{n}}$, the numerator of the exponent is a “power”, and the denominator represents a “root”.

e.g. For the number $16^{\frac{3}{4}}$

$$\begin{aligned} 16^{\frac{3}{4}} &= \left(16^{\frac{1}{4}}\right)^3 \\ &= (2)^3 \\ &= 8 \end{aligned}$$

separate the numerator from the denominator

$$(16^3)^{\frac{1}{4}}$$

*↑
makes the root part too big*

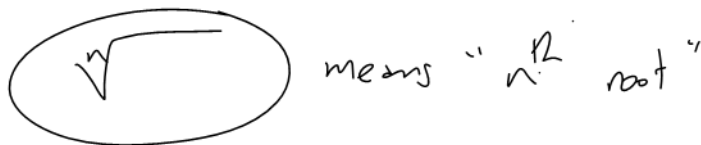
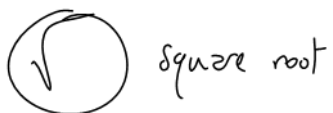
Note:

$$16^{\frac{1}{4}} \text{ MEANS}$$

some number "a" has the property $a^4 = 16$

$$\Rightarrow a = 2$$

Always evaluate the "root part" first.



Example 4.3.1

From your text: Pg. 229 #2.

Write in exponent form, and then evaluate:

a) $\sqrt[3]{512}$

$$= (512)^{\frac{1}{3}}$$

$$= 8$$

c) $\sqrt[3]{27^2}$

$$= (27^2)^{\frac{1}{3}}$$

$$= (27^{\frac{1}{3}})^2$$

$$= 3^2$$

$$= 9$$

b) $\sqrt[3]{-27}$

$$= (-27)^{\frac{1}{3}}$$

$$= -3$$

Note: We **CANNOT** take an **Even Root** of a **negative radicand**.

We **CAN** take an **Odd Root** of a negative radicand, however.

f) $\sqrt[4]{\left(\frac{16}{81}\right)^{-1}}$

$$= \left(\left(\frac{16}{81} \right)^{-1} \right)^{\frac{1}{4}}$$

$$= \left(\frac{81}{16} \right)^{\frac{1}{4}} = \frac{81^{\frac{1}{4}}}{16^{\frac{1}{4}}} = \frac{3}{2}$$

Example 4.3.2

From your text: Pg. 229 #3

Write as a single power:

a) $\left(8^{\frac{2}{3}}\right)\left(8^{\frac{1}{3}}\right)$

$$= 8^{2/3 + 1/3}$$

$$= 8^1$$

$$= 8$$

b) $8^{\frac{2}{3}} \div 8^{\frac{1}{3}}$

$$= 8^{2/3 - 1/3}$$

$$= 8^{\frac{1}{3}}$$

$$= 2$$

Example 4.3.3

From your text: Pg. 229 #4

Write as a single power, then **evaluate**. Express answers in **rational form**.

$$\begin{aligned}
 \text{a) } \sqrt{5} \cdot \sqrt{5} &= (5)^{\frac{1}{2}} \cdot (5)^{\frac{1}{2}} \\
 &= 5^{\frac{1}{2} + \frac{1}{2}} \\
 &= 5^1 = 5
 \end{aligned}
 \qquad
 \begin{aligned}
 \text{b) } \frac{\sqrt[3]{-16}}{\sqrt[3]{2}} &= \frac{(-16)^{\frac{1}{3}}}{(2)^{\frac{1}{3}}} \\
 &= \left(\frac{-16}{2} \right)^{\frac{1}{3}} \\
 &= (-8)^{\frac{1}{3}} \\
 &= -2
 \end{aligned}$$

Class/Homework: Pg. 229 #2de, 3cdef, 4cd, 5, 6, 8

10 (a question of awesomeness)

12 (we may take up next day)