Chapter 4 – Exponential Functions

4.5 – 4.6 – Properties and Transformations of Exponential Functions

Exponential Functions are of the (basic) form:

$$f(x) = b^{x}$$

exponent is variable

$f(x) = x^3$

b is the base

(of course, we can apply transformations to this basic, or parent, function!! Fun Times are – a – coming!!)

In the basic exponential function $f(x) = b^x$, b is the base. **The base of an exponential function is just a number**. For example, we might have the functions

$f(x) = 2^x$ or $g(x) = 3^x$ or $h(t) = \left(\frac{1}{4}\right)^t$

base 2 base 3 base $\frac{1}{4}$

What the Base of an Exponential Function tells you

whether the function is describing a growth or a decay phenomenon.

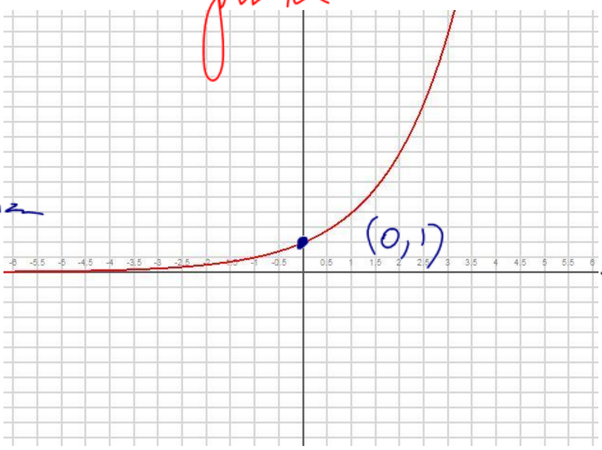
Note: we take "b" (the base) to be a positive number and $b \neq 1$

Domain and Range of the Basic Exponential Function

Consider the sketches:

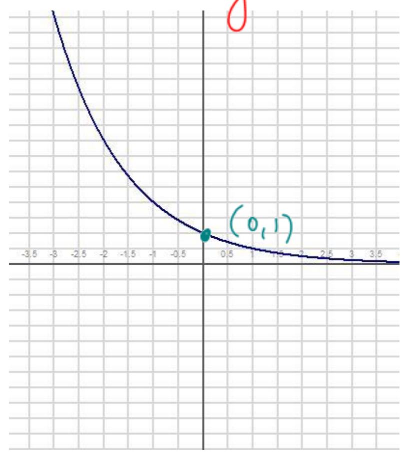
growth

base is greater than 1



y = 0 is a Horizontal Asymptote.

decay



base is a fraction between 0 and 1

$f(x) = 2^x$

$D_f: \{x \in \mathbb{R}\}$.

$R_f: \{f(x) \in \mathbb{R} \mid f(x) > 0\}$

$g(x) = \left(\frac{1}{2}\right)^x$

$D_g = \{x \in \mathbb{R}\}$.

$R_g = \{g(x) \in \mathbb{R} \mid g(x) > 0\}$

ALL Exponential Functions have a Horizontal **ASYMPTOTE** (Basic Exponential Functions have $y = 0$ as their Horizontal Asymptote. The Horizontal Asymptote of a Transformed Exponential Function depends on the vertical shift.

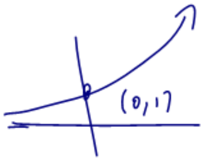
The H.A. will move w/ the vertical shift.

ALL BASIC Exponential Functions pass through the point $(0,1)$. ← y-int

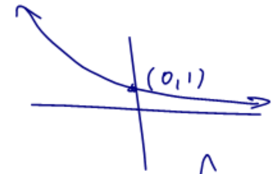
↳ NON TRANSFORMED

Transformed Exponential Functions will have a y-intercept, but depends on a number of transformations

1) Vertical Shifts
2) Vertical stretches and flips
3) Horizontal Shifts



$f(x) = 2^x$



$f(x) = 2^{-x}$

$= \left(\frac{1}{2}\right)^x$

The Transformed Exponential Function

The general form of an exponential function is:

$$f(x) = a \cdot b^{k(x-d)} + c$$

Where:

a = vertical stretch

k = horizontal stretch (factor $\frac{1}{k}$)

c = vertical shift

d = horizontal shift (right d units)

Horizontal Asymptote $y = c$

Example 4.6.1

State the transformations applied to the parent function $f(x) = 3^x$. Also state the y-intercept, and the equation of the horizontal asymptote of the transformed function.

$g(x) = -2 \cdot 3^{3x+3} + 4 = -2 \cdot 3^{3(x+1)} + 4$

H.A.

$y = 4$

vertical

Flip: yes

Stretch: x2

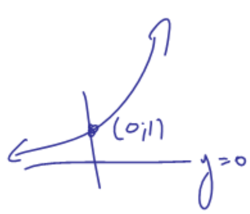
Shift: up 4

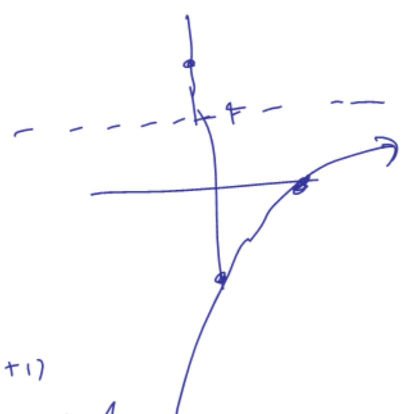
horizontal

Flip - No

Stretch: $\frac{1}{3}$

Shift: 1 left





y int : $x = 0$

$g(0) = -2 \cdot 3^{3(0+1)} + 4$

$= -2(3^3) + 4$

$= -50$

(0, -50)

Example 4.6.2

From your Text: Page 252 #7

of the liquid.

Horizontal Asymptote $y = 20$

7. A cup of hot liquid was left to cool in a room whose temperature was 20 °C. The temperature changes with time according to the function, t in minutes $T(t) = 80\left(\frac{1}{2}\right)^{\frac{t}{30}} + 20$. ~~Use your knowledge of transformations~~ to sketch this function. Explain the meaning of the y-intercept and the asymptote in the context of this problem.

time is "chunked" into 30 minutes

base $\frac{1}{2}$ means "decay" (cooling)

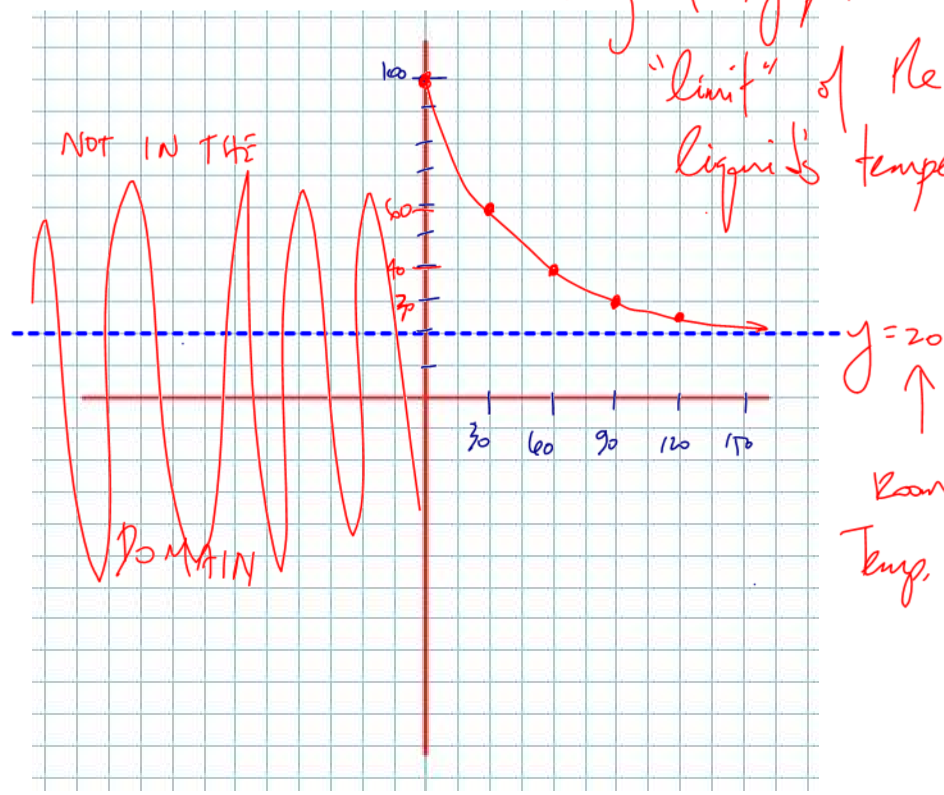
- The y-int is the initial Temperature
- The Horizontal Asymptote is the "limit" of the liquid's temperature

y-int: $t = 0$

$T(0) = 80\left(\frac{1}{2}\right)^{\frac{0}{30}} + 20$

$= 100°C$

$\Rightarrow (0, 100)$



$T(30) = 80\left(\frac{1}{2}\right)^{\frac{30}{30}} + 20$

$= 60°$ $(30, 60)$

$T(60) = 80\left(\frac{1}{2}\right)^{\frac{60}{30}} + 20$

$= 80\left(\frac{1}{2}\right)^2 + 20$

$= 80\left(\frac{1}{4}\right) + 20$

$= 40°C$

$T(90) = 80\left(\frac{1}{2}\right)^{\frac{90}{30}} + 20$

$= 80\left(\frac{1}{2}\right)^3 + 20$

$= 80\left(\frac{1}{8}\right) + 20$

$= 30°$

$T(120) = 80\left(\frac{1}{2}\right)^{\frac{120}{30}} + 20$

$= 80\left(\frac{1}{2}\right)^4 + 20$

$= 80\left(\frac{1}{16}\right) + 20$

$= 25°C$

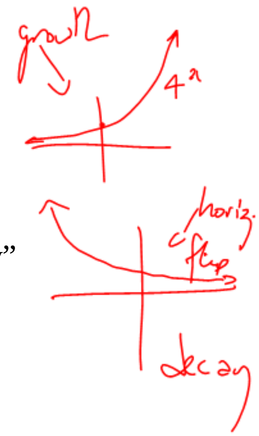
Example 4.6.3

From your Text: Page 252 #5a

growth base $4 > 1$

Let $f(x) = 4^x$. For the function which follows,

- State the transformations applied to $f(x)$
- State the y-intercept, and the horizontal asymptote
- Sketch the transformed function, and write the function "properly"
- State the domain and range of the transformed function



$$g(x) = 0.5 f(-x) + 2$$

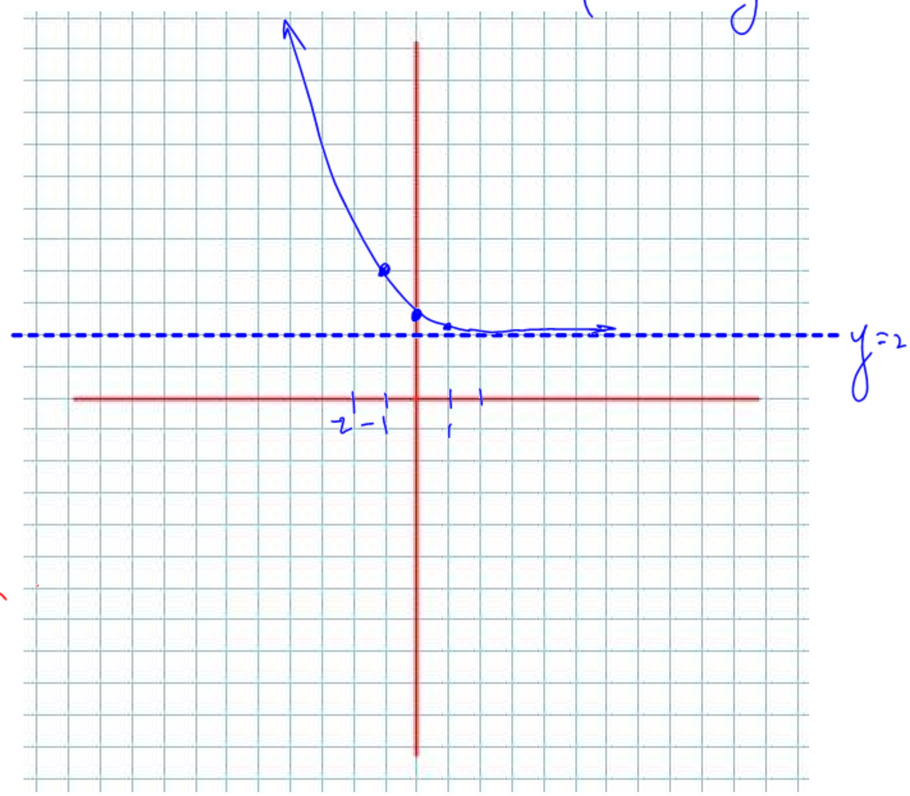
VST S

$$g(x) = 0.5\left(4^{-x}\right) + 2$$

$$= 0.5\left(4^{x}\right)^{-1} + 2$$

$$= 0.5\left(\frac{1}{4}\right)^{x} + 2$$

3 points of accuracy



Vertical Stretch $\times \frac{1}{2}$

Shift up 2

Horizontally Flip (transforms growth to decay)

y-int: $g(0) = 0.5\left(\frac{1}{4}\right)^0 + 2$

$= 2.5$

$(0, 2.5)$

H.A. $y = 2$

$g(1) = 0.5\left(\frac{1}{4}\right)^1 + 2$

$= 2.125$ $(1, 2.125)$

$g(-1) = 0.5\left(\frac{1}{4}\right)^{-1} + 2$

$= 4$ $(-1, 4)$

Class/Homework: Pg. 251 – 253 #1 – 3, 5bcd (write each transformed function "properly), 8 – 10 (for #10 please see example 4 on page 250)