Chapter 4 – Exponential Functions

4.7 – Applications of Exponential Functions

Anything in the real world which grows, or decays can be "**modeled**" (or in some sense **"described"**) with words, or pictures or mathematics. Mathematical **models** are useful for getting solutions to problems, and making predictions.

So far in Mathematics 11U we have studied the basics of functions in general (chapter 1), we've done some algebra (chapter 2), and we've examined Quadratic functions (chapter 3). Part of our study of Quadratics was learning how to use the vertex of a parabola to answer questions about maxima and minima for some real word problems. For example we saw a question where we tried to maximize revenue for a school store. Quadratic **models** are very useful for solving max/min problems.

In this lesson we want to work on **learning how to solve problems dealing with growth and decay.** We have to decide what type of function will best model (or describe) the type of growth/decay seen in the problem (hint: for this lesson we'll be examining Exponential Growth and Decay, and therefore we expect that exponential functions will be used…shocking, I know)

Q. What is Exponential Growth or Decay?

Consider the following:

A single cell divides into two "daughter" cells. Both daughter cells divide resulting in four cells. Those four cells each divide and we now have a population of 8

Describe, using mathematics, how the cell population changes from generation to generation.

Gen 0	Gen 1	Gen 2	Gen 3	Gen 4	Gen 5
1	2	4	8	16	32

$f(x) = 2^x$, x = generation # This works

Example 4.7.1

Being a financial wizard, you deposit \$1 000 into an account which pays 3.5% interest, annually.

a) Determine ~~who~~ how much money is in your account after $t = 1,2,\cancel{3}$, and $\cancel{4}$ years.

b) Determine a mathematical model which can describe how the value of the account is changing from year to ~~p~~ year.

years 0 1 2

\$1000 (\$1000 + Interest) (1035 + Interest)

$* = (\$1000 + 0.035(1000))$ $* = (1035 + (1035)(0.035))$

$= (\$1035)$ $= \$1071.25$

$A_0 = \$1000$

$r = \text{rate}$

A_n : Amount after n years

$$\boxed{A_1 = A_0 + A_0 r = A_0(1+r)}$$

$A_2 = A_1 + A_1 r$

$= \boxed{A_1}(1+r)$

$= \boxed{(A_0(1+r))}(1+r)$

$= A_0(1+r)^2$

$A_3 = A_0(1+r)^3$

$A_{10} = A_0(1+r)^{10}$

$$\boxed{A_n = A_0(1+r)^n}$$

Exponential

Definition 4.7.1

A function describing Exponential Growth is of the form:

$$A(t) = A_0(1+r)^t$$

where r is the growth rate. $0 < r < 1$

A function describing Exponential Decay is of the form:

$$A(t) = A_0(1-r)^t$$

constant given constant given

$0 < r < 1$

r is called the decay rate.

Example 4.7.2

From your text, Pg. 263

10. In each case, write an equation that models the situation described. Explain what each part of each equation represents.

a) the percent of colour left if blue jeans lose 1% of their colour every time they are washed

decay

b) the population if a town had 2500 residents in 1990 and grew at a rate of 0.5% each year after that for *t* years

c) the population of a colony if a single bacterium takes 1 day to divide into two; the population is *P* after *t* days

a) $C(\omega) = C_0(1-r)^{\omega}$ $C_0 = 1$ $r = 0.01$

$C(\omega) = 1(1-0.01)^{\omega}$

$= 0.99^{\omega}$

b) $P(t) = P_0(1+r)^t$ $P_0 = 2500$ 0.5% → $r = 0.005$

$\Rightarrow P(t) = 2500(1.005)^t$ Note: $t = 0$ is 1990

c) $P(t) = P_0(1+r)^t$ $P_0 = 1$ $r = 1$

$\Rightarrow P(t) = 1(2)^t = 2^t$

Example 4.7.3

From your text, Pg. 263

11. A population of yeast cells can double in as little as 1 h. Assume an initial population of 80 cells.

a) What is the growth rate, in percent per hour, of this colony of yeast cells?

b) Write an equation that can be used to determine the population of cells at *t* hours.

c) Use your equation to determine the population after 6 h.

d) Use your equation to determine the population after 90 min.

e) Approximately how many hours would it take for the population to reach 1 million cells? $P(t)$, $t =$ unknown.

f) What are the domain and range for this situation?

Variables

$P(t)$ final value

t (hours) domain

Constants

$P_0 = 80$

$r = 1$

b) $P(t) = 80(2)^t$

c) $P(6) = 80(2)^6$

$= 5120$ cells

90 mins = 1.5 hrs

d) $P(1.5) = 80(2)^{1.5}$

$= 226$ cells round down

e) $1\,000\,000 = 80(2)^t$

$\Rightarrow \frac{1\,000\,000}{80} = 2^t$

$\rightarrow 2^t = 12\,500 \quad = \quad t = \frac{\log(12500)}{\log(2)}$

$2^{(13.6)} = 12416\ldots$

$= 13.6$ hours

Class/Homework:

READ Example 2 on pages 256 – 257 (which method do you prefer: Guess and Check, or Graphing Calculator?)

READ Example 4 on pages 259 – 260.

Pg. 261 – 263 #1, 3 – 9, 12 – 16 (you have two days for these problems)

"log"