Chapter 4 – Exponential Functions

Additional Review – Doubling and Half-Life

Example (Doubling)

P_0 $t=0$ (noon) "chunked time"

6. A species of bacteria has a population of 500 at noon. It doubles every 10 h.
K The function that models the growth of the population, P, at any hour, t, is

$$P(t) = 500\left(2^{\frac{t}{10}}\right).$$ doubling

$$\boxed{P(t) = P_0(1+r)^{t/D}}$$

$r = 1$

D = doubling period.

a) Why is the exponent $\frac{t}{10}$? , we chunk time into "doubling periods"
b) Why is the base 2?
c) Why is the multiplier 500?
d) Determine the population at midnight.
e) Determine the population at noon the next day.
f) Determine the time at which the population first exceeds 2000.

b) because we are doubling

c) $P_0 = 500$ is the population of bacteria when measuring of the population begins

d) Midnight noon is $t=0$

" $t = 12$

$$P(12) = 500\left(2^{\frac{12}{10}}\right)$$

$$= 500\left(2^{1.2}\right) = 1148 \text{ bacteria.}$$

e) $t = 24$

$$P(24) = 500\left(2^{\frac{24}{10}}\right) = 500\left(2^{2.4}\right) = 2639 \text{ bacteria}$$

f) we want t when $P(t) = 2000$

$$2000 = 500\left(2^{t/10}\right)$$

$$\Rightarrow 4 = 2^{t/10}$$

base 2!

$$2^2 = 2^{t/10}$$

$$\Rightarrow 2 = t/10$$

$$\therefore\ t = 20 \text{ hours}$$

recall

$$a^b = a^c$$

$$\Rightarrow b = c$$

$$4 = 2^{t/10}$$

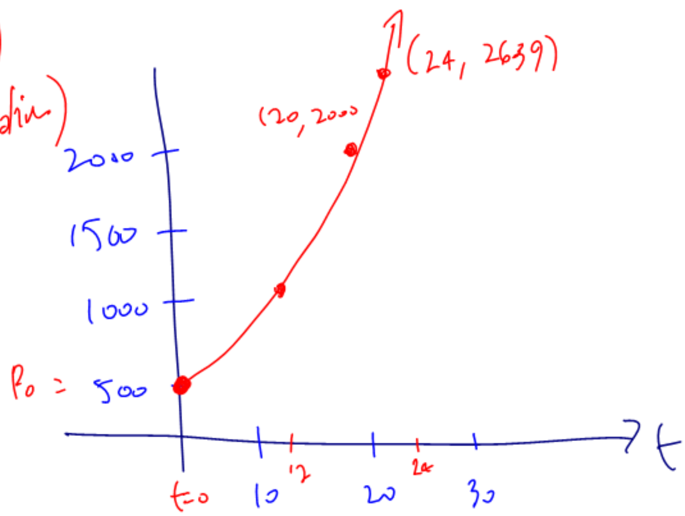
$$\Rightarrow \log(4) = \log\left(2^{t/10}\right)$$

$$\Rightarrow \frac{t}{10} = \frac{\log(4)}{\log(2)}$$

$$\Rightarrow t = 10\left(\frac{\log(4)}{\log(2)}\right)$$

(t, P(n))

(time, population)



amount of time for $\frac{1}{2}$ of the original amount to decay to $\frac{1}{2}$ of the original amount

Example (Half-Life)

9. A student records the internal temperature of a hot sandwich that has been left to cool on a kitchen counter. The room temperature is 19 °C. An equation that models this situation is

$$T(t) = 63(0.5)^{\frac{t}{10}} + 19$$

where T is the temperature in degrees Celsius and t is the time in minutes.

a) What was the temperature of the sandwich when she began to record its temperature?

b) Determine the temperature, to the nearest degree, of the sandwich after 20 min.

c) How much time did it take for the sandwich to reach an internal temperature of 30 °C?

general eqn

$$A(t) = A_0\left(\frac{1}{2}^{t/h}\right)$$

h = half-life.

a) $T(0) = 63(0.5)^{0/10} + 19$

$= 82°C$ ⇒ point $(0, 82)$ (time, temperature)

b) $T(20) = 63(0.5)^{20/10} + 19$

$= 63(0.5^2) + 19$

$= 34.75°C \Rightarrow 35°C$

c) We want time when Temp is 30°

$30 = 63(0.5)^{t/10} + 19$

$\Rightarrow 11 = 63(0.5^{t/10})$

$\Rightarrow \frac{11}{63} = 0.5^{t/10}$

$\Rightarrow \boxed{0.1746 = 0.5^{t/10}}$

SAMDEB

Guess & Check.

try $t = 25$

$0.5^{2.5} = 0.1768$

$t = 26$: $0.5^{2.6} = 0.1649$

log method

$$0.1746 = 0.5^{t/10}$$

$$\log(0.1746) = \frac{t}{10}\left(\log(0.5)\right)$$

$$\Rightarrow \frac{t}{10} = \frac{\log(0.1746)}{\log(0.5)}$$

$$\Rightarrow t = 10\left(\frac{\log(0.1746)}{\log(0.5)}\right) = 25.18 \text{ minutes}$$

$t = 25.5$ min

$0.5^{2.55} = 0.1707$

25.5 mins gives 2 decimal places of accuracy