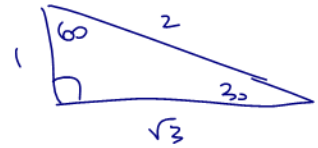
Hwk Check

pg 287

Prove

9. Show that $\tan 30° + \dfrac{1}{\tan 30°} = \dfrac{1}{\sin 30° \cos 30°}$.



LHS

$\tan(30) + \dfrac{1}{\tan(30)}$

$= \dfrac{1}{\sqrt{3}} + \dfrac{\sqrt{3}}{1} = \dfrac{1}{\sqrt{3}} + \dfrac{\sqrt{3}}{1} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{1+3}{\sqrt{3}} = \dfrac{4}{\sqrt{3}}$

RHS

$\dfrac{1}{\sin(30)\cos(30)}$

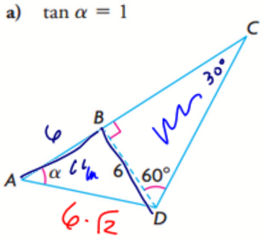
$= \dfrac{1}{\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)} = \dfrac{\frac{1}{1}}{\frac{\sqrt{3}}{4}}$

$= \dfrac{4}{\sqrt{3}} = \text{LHS}$

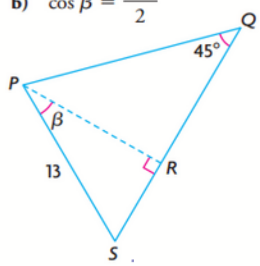
$\square$

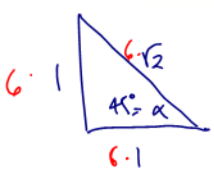
11. Determine the exact area of each large triangle.

a) $\tan \alpha = 1$



b) $\cos \beta = \dfrac{\sqrt{3}}{2}$

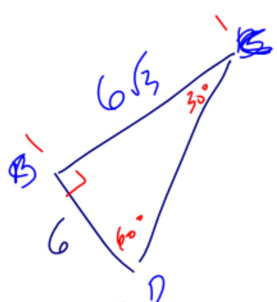


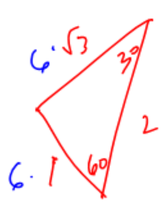


area $\triangle ABD$

$= \frac{1}{2}(6)(6)$

$= 18$





Area of $\triangle BCD$

$= \frac{1}{2}(6)(6\sqrt{3})$

$= 18\sqrt{3}$

$\therefore$ Area of $\triangle ACD$

$= 18 + 18\sqrt{3} = 18(1+\sqrt{3})$

Chapter 5 – Trigonometric Ratios

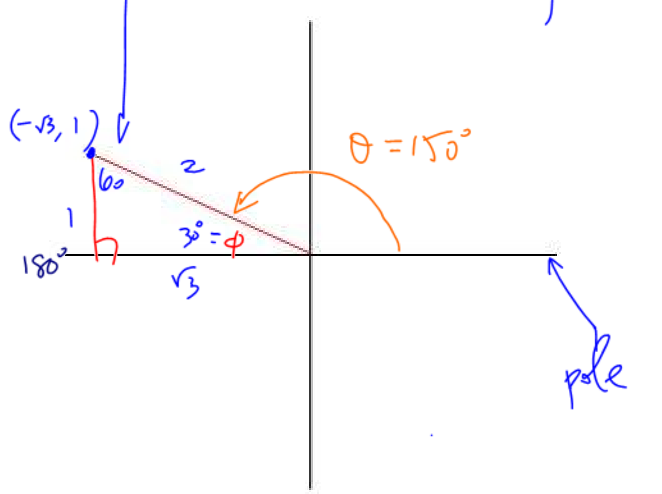
5.3 – 5.4 – Trigonometric Ratios for Angles Lager than 90º

Angles Larger than 90º

Consider the following sketch of the angle $\theta = 150^\circ$

terminal arm for the angle of rotation

θ is an angle of rotation



ϕ = phi = 30°

e.g. Calculate

$\sin(150) = 0.5$

$\sin(30) = 0.5$

$\cos(150) = -0.866$

$\cos(30) = +0.866$

$\tan(150) = -0.577$

$\tan(30) = 0.577$

sin(150) makes no sense in terms of SOH CAH TOA no right angle Δ with 150°

WHAT IS GOING ON ?

However, we can construct a right angle Δ by dropping a ⊥ to THE POLE

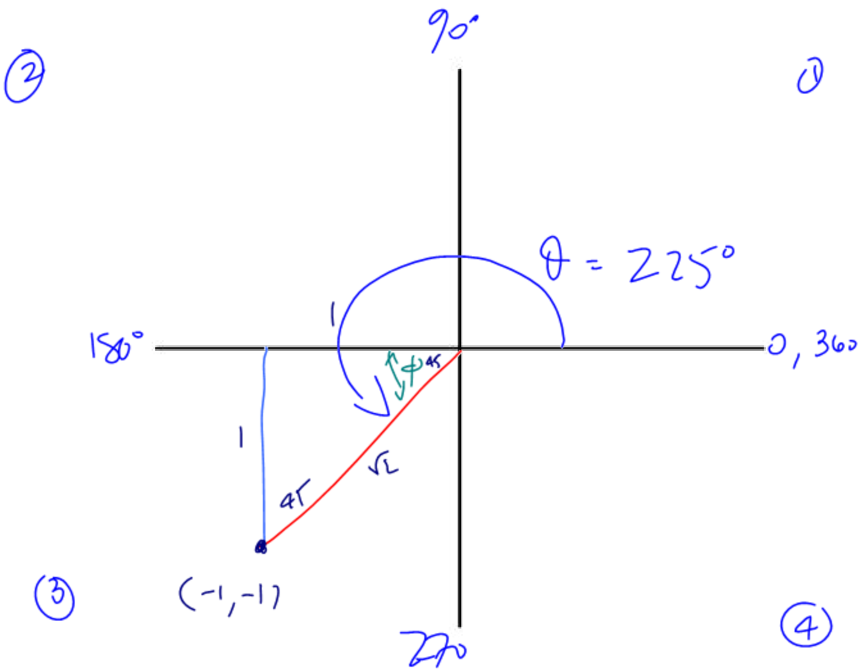
In this example, we call $\theta = 150^\circ$ the **PRINCIPAL ANGLE**, or the angle in STANDARD POSITION

Note: The angle $\phi = 30^\circ$ is called the
RELATED ACUTE ANGLE

Example 5.3.1

Sketch the angle of rotation $\theta = 225^\circ$ and determine the related acute angle.

$\phi = 225 - 180$
$= 45^\circ$



e.g. Calculate

$\sin(225) = -0.707$

$\sin(45) = +0.707$

$\cos(225) = -0.707$

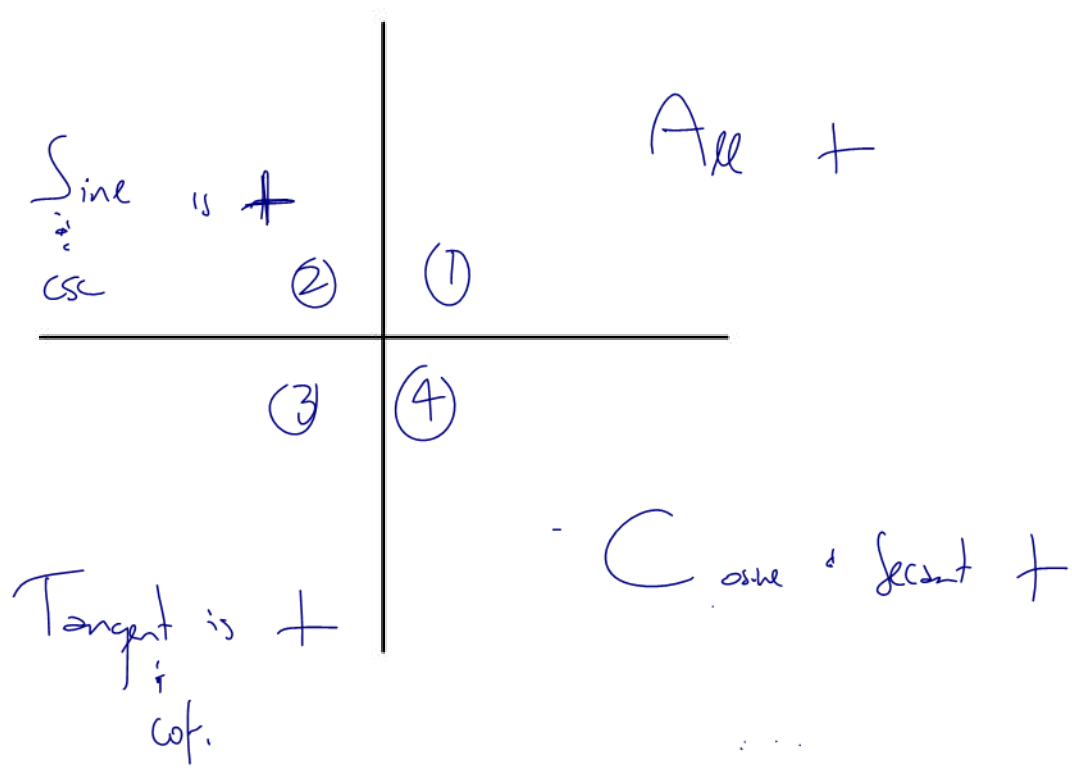
$\cos(45) = +0.707$

$\tan(225) = +1$

$\tan(45) = +1$

What is up with these signs??? (Be Careful with your signs!!!!!!!!!)

The **CAST RULE** determines the sign (+ or -) of the trig ratio



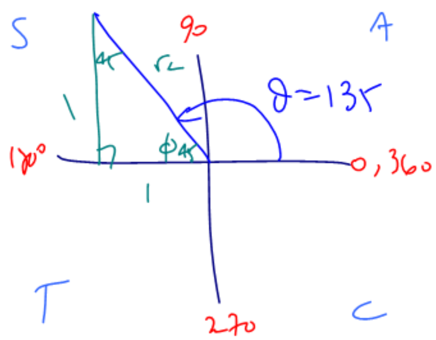
We now have enough tools to calculate the trigonometric ratios of any angle!

For any given angle θ we will:

1) Draw θ in standard position (i.e. draw the principal angle for θ)
2) Determine the related acute angle (between the terminal arm and the x-axis (also called the polar axis)
3) Use the related acute angle and the CAST RULE (and SOH CAH TOA) to determine the trig ratio (along with its sign…BE CAREFUL WITH YOUR SIGNS) in question

Example 5.3.2

Determine the trig ratio $\sin(135)$ exactly



$\phi = 180 - 135$
$= 45°$

$\cos(135) = -\frac{1}{\sqrt{2}}$

$\sin(135) = +\frac{1}{\sqrt{2}}$

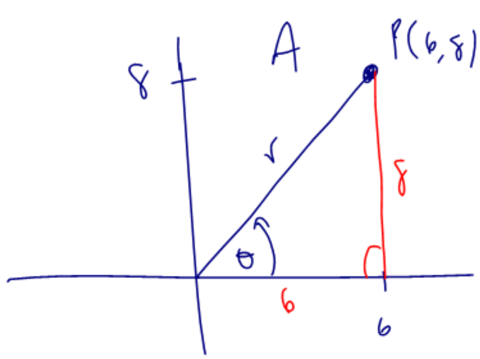
$\tan(135) = -1$

Example 5.3.3

The point $P(x, y) = (6,8)$ lies on the terminal arm (of length *r*) of an angle of rotation.

Sketch the angle of rotation.

Determine: a) the value of *r*
b) the primary trig ratios for the angle
c) the value of the angle of rotation in degrees, to two decimal places



a) $r = \sqrt{6^2 + 8^2}$
$= 10$

b) $\sin(\theta) = +\frac{8}{10} = \frac{4}{5}$

$\cos(\theta) = +\frac{6}{10} = \frac{3}{5}$

$\tan(\theta) = +\frac{8}{6} = \frac{4}{3}$

c) $\tan(\theta) = \frac{4}{3}$

$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53°$

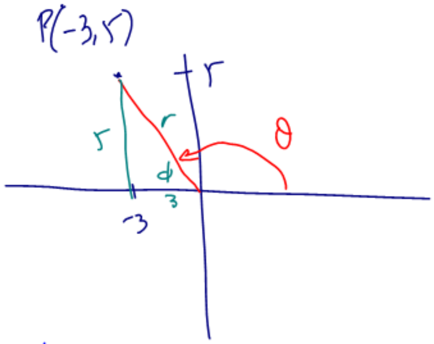
Example 5.3.4

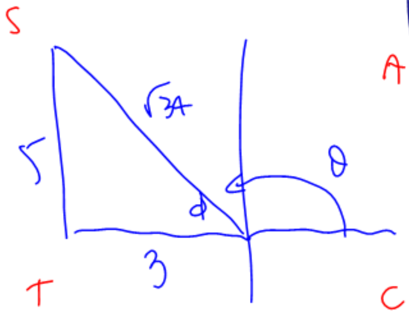
The point $(-3, 5)$ lies on the terminal arm (of length r) of an angle of rotation. Sketch the angle of rotation.

Determine: a) the value of r

b) the primary trig ratios for the angle

c) the value of the angle of rotation in degrees, to two decimal places





a) $r = \sqrt{3^2 + 5^2}$

$= \sqrt{9+25}$

$= \sqrt{34}$

b) $\sin(\theta) = +\frac{5}{\sqrt{34}}$

$\cos(\theta) = -\frac{3}{\sqrt{34}}$

$\tan(\theta) = -\frac{5}{3}$

c) $\theta = 121°$

$(180 - 59°)$

$\theta = \sin^{-1}\left(\frac{5}{\sqrt{34}}\right)$

$\approx 59°$

$\theta = \cos^{-1}\left(-\frac{3}{\sqrt{34}}\right)$

$= 121°$

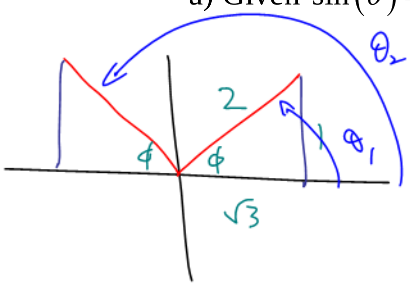
$\theta = \tan^{-1}\left(-\frac{5}{3}\right)$

$= -59°$

clockwise

Example 5.3.5 (going backwards!)

a) Given $\sin(\theta) = +\frac{1}{2}$ determine BOTH values of θ for $0° \le \theta \le 360°$



$\phi = 30°$

$\theta_1 = 30°$

$\theta_2 = 180 - 30°$

$= 150°$

−ve in Q2 and Q3

b) Given $\cos(\theta) = -0.5372$ determine BOTH values of θ for $0° \le \theta \le 360°$

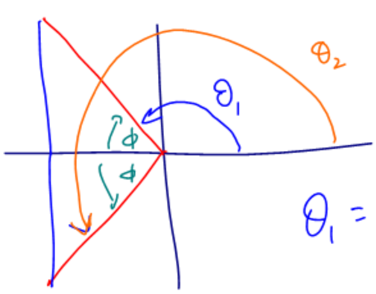
Algorithm

(1) Determine the quadrants θ is in

(2) If there is a negative - ignore it!

(3) calculate $\cos^{-1}(\text{ratio}) = \phi$

(4) use φ to find θ



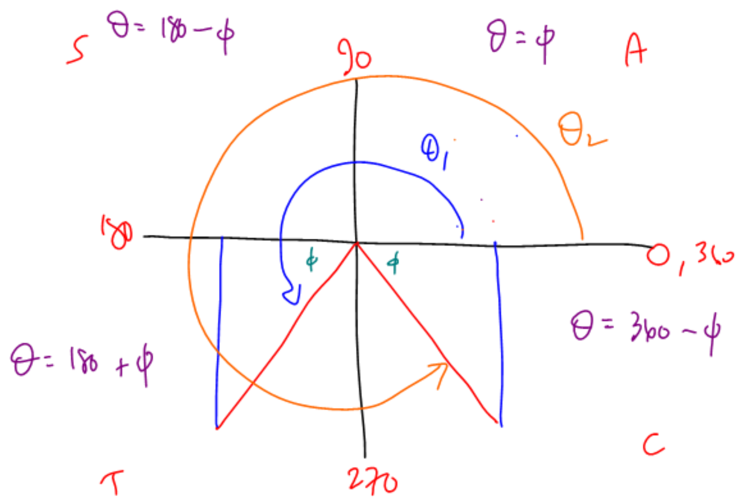
$\phi = \cos^{-1}(0.5372)$

$= 58°$

$\theta_1 = 180 - 58 = 122°$

$\theta_2 = 180 + 58 = 238°$

c) Given $\sin(\theta) = -0.4567$ determine BOTH values of θ for $0^\circ \le \theta \le 360^\circ$



$\phi = \sin^{-1}(0.4567)$

$= 27^\circ$

$\theta_1 = 180^\circ + 27^\circ = 207^\circ$

$\theta_2 = 360^\circ - 27 = 333^\circ$

Class/Homework

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