Chapter 5 – Trigonometric Ratios

5.6: The Sine Law

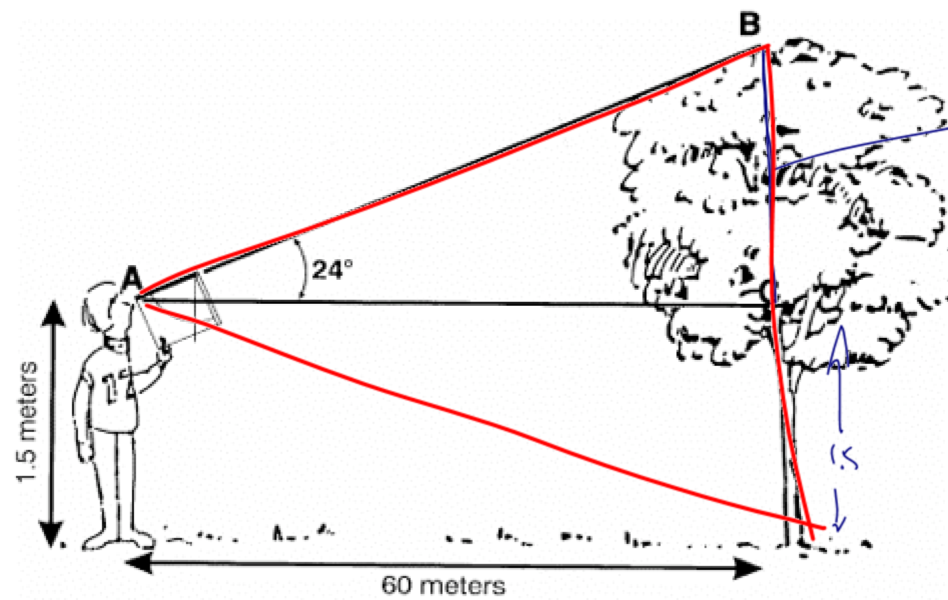
Last year you learned the Sine Law. It is a “formula” we can use to **solve triangles which are not right angle triangles**. There is one requirement to be able to use the Sine Law.

You Must Have an Angle With Its Corresponding Side!

So far we have been using Right Angle Triangles along with SOH CAH TOA to “solve” triangles. BUT right angle triangles aren’t always the best triangle to use;

Consider the following picture:

Using a Right Angle Triangle has its issues…



$$\tan(24) = \frac{\overline{BC}}{60}$$

$$\Rightarrow \overline{BC} = (60)(\tan 24)$$

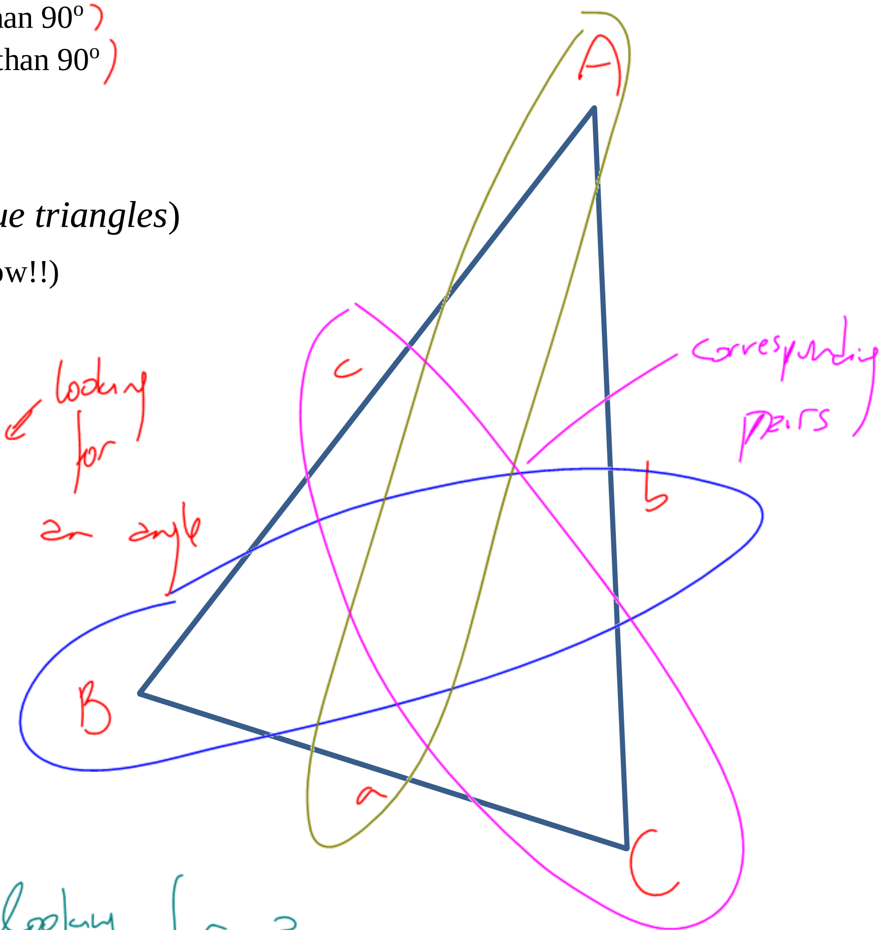
Sometime using a right angle triangle just can't be done. We then need to use so-called "**OBLIQUE TRIANGLES**". Oblique triangles come in two forms:

1) Acute (all angles are less than 90^o)
2) Obtuse (one angle is more than 90^o)

The Sine Law (*for oblique triangles*)

(There are **TWO FORMS** you should know!!)

Given the non-right triangle, ΔABC, then:



looking for an angle

$$\frac{\sin(A)}{a}=\frac{\sin(B)}{b}=\frac{\sin(C)}{c}$$

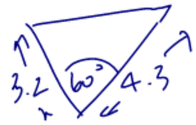
or

$$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$$

looking for a side length

Notes:

1) Memorize the SINE LAW!
2) If we are trying to **find an angle**, use the first form of the Sine Law (**angles on top**)
3) If we are trying to **find the length** of a side, use the second for of the law (**with sides on top**)
4) In order to use the Sine Law, you must have the correct information in the triangle. You must have:
    a) 3 pieces of information
    b) One "**CORRESPONDING PAIR**" – an angle with its opposite side (for example you might have side a and angle A)



SAS

ASS

Note: **There is a problem with the Sine Law**

Recall that for trig ratios, "sine" is positive in quadrants 1 ***and*** 2.

e.g. $\sin\left(51^{\circ}\right) =$ 0.777 $\qquad \sin\left(129^{\circ}\right) =$ 0.777

$\sin(\theta) = 0.777$

$\theta = \sin^{-1}(0.777)$

$= 51^{\circ}$

Q/ What about 129°?

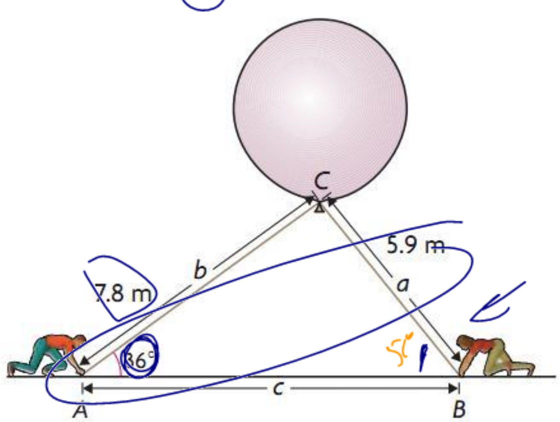
Consider Example 1 in your text: Pg. 312 – 314 .

Albert and Belle are part of a scientific team studying thunderclouds. The team is about to launch a weather balloon into an active part of a cloud. Albert's rope is 7.8 m long and makes an angle of 36° with the ground. Belle's rope is 5.9 m long.

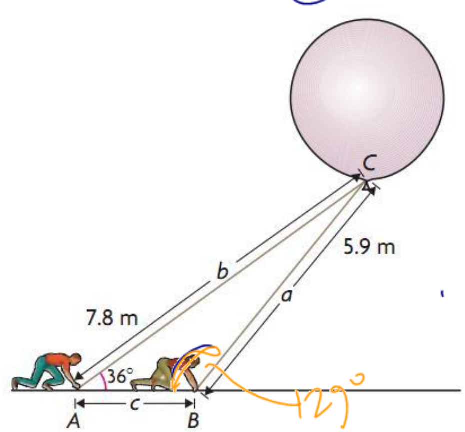
The question we are asked is: **How far is Albert from Belle, to the nearest meter?**

Possible Pictures:

1



2



Both pictures describe the problem completely. So which is correct? Well…BOTH ARE **POSSIBLE** solutions. This is known as the "**AMBIGUOUS CASE**". Because both are possible solutions, you must find both.

Note: If the **GIVEN ANGLE** is **ACUTE**, then this so called Ambiguous Case MAY APPLY. But, if the Given Angle is Obtuse, then the Ambiguous Case CANNOT APPLY. (*And* ***Sometimes****, there is no triangle which solves the problem.*)

Why?

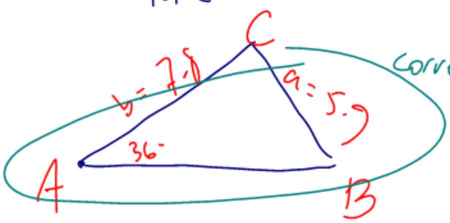
θ cannot be obtuse

find (state all 6 bits of $\triangle$ info)

Example 5.6.1

Solve the triangles above.

Picture



Answers

reasons

$\angle A = 36°$ (g) $\quad a = 5.9\text{ m}$ (g)

$\angle B = 51°$ (S.L.) $\quad b = 7.8\text{ m}$ (g)

$\angle C = 93°$ (+) $\quad c = 10$ (SL)

$180 - (36 + 51)$

(angle-sum-triangle theorem)

$\boxed{ASTT}$

(angles add to 180)

Work

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

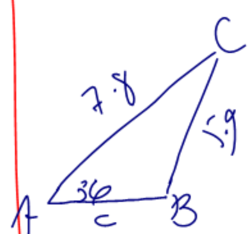
$$\frac{\sin B}{7.8} = \frac{\sin(36)}{5.9}$$

$$\sin(B) = \frac{(7.8)\sin(36)}{5.9}$$

$$\angle B = \sin^{-1}\left(\frac{(7.8)(\sin(36))}{5.9}\right)$$

$$= 51°$$

BUM CASE



$\angle A = 36°$ (g) $\quad a = 5.9$ g

$\angle B = 129°$ (BC) $\quad b = 7.8$ (g)

$180 - 51 = 129$

$\angle C = 15°$ (ASTT) $\quad c = 2.6$ (S.L)

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$c = \frac{(5.9)(\sin(15))}{\sin(36)} = 2.6$$

$$\frac{c}{\sin(C)} = \frac{a}{\sin(A)}$$

$$\frac{c}{\sin(93)} = \frac{5.9}{\sin(36)}$$

$$c = \frac{(5.9)(\sin(93))}{\sin(36)}$$

$$= 10\text{ cm.}$$

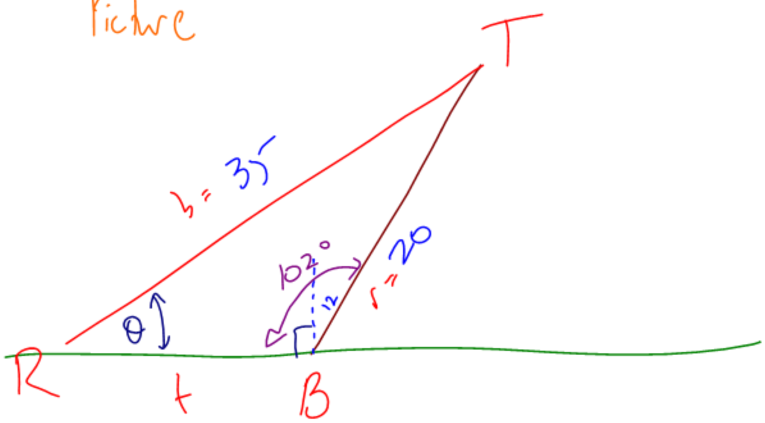
∴ Albert & Belle are either 10 m or 2.6 m apart

Example 5.6.2

From your text: Pg. 319 #6

The trunk of a leaning tree makes an angle of 12° with the vertical. To prevent the tree from falling over, a 35.0 m rope is attached to the top of the tree and is pegged into level ground some distance away. If the tree is 20.0 m from its base to its top, calculate the angle the rope makes with the ground to the nearest degree.

Picture



$$\boxed{\frac{\sin(\theta)}{r} = \frac{\sin(B)}{b}}$$

$$\sin(\theta) = \frac{(20)(\sin(102))}{35}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{20(\sin(102))}{35}\right)$$

$$= 34°$$

$\therefore$ The rope makes an angle of 34°

Example 5.6.3

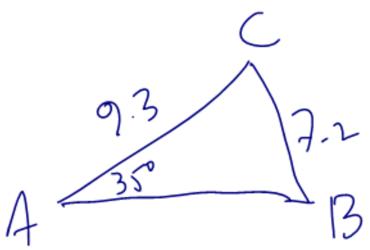
From your text: Pg. 318 #5ac

Where appropriate, sketch all possible triangles, given each set of information. Label all side lengths to the nearest tenth of a centimetre and all angles to the nearest degree.

a) $a = 7.2$ mm, $b = 9.3$ mm, $\angle A = 35°$

c) $a = 1.3$ cm, $b = 2.8$ cm, $\angle A = 33°$

a)



$\angle A = 35°$ (g) $\quad a = 7.2$ (g)

$\angle B = 48°$ (SL) $\quad b = 9.3$ (g)

$\angle C = 97°$ (ASTT) $\quad c = 12.5$ mm (SL)

$$\frac{\sin B}{9.3} = \frac{\sin(35)}{7.2}$$

$$\frac{c}{\sin(C)} = \frac{a}{\sin(A)}$$

$$c = \frac{(7.2)\sin(97)}{\sin(35)} = 12.5 \text{ mm}$$

$$B = \sin^{-1}\left(\frac{(9.3)(\sin(35))}{7.2}\right)$$

$$= 48°$$

Bom case

$\angle A = 35°$ (g) $\quad a = 7.2$ (g)

$\angle B = 132°$ (BC) $\quad b = 9.3$ (g)

$\angle C = 13°$ (+) $\quad c = 2.8$ (SL)

$$\frac{c}{\sin(13)} = \frac{7.2}{\sin(35)} \Rightarrow c = \frac{(7.2)(\sin(13))}{\sin(35)}$$

$$= 2.8$$

Class/Homework: Pg. 318 – 320 #4, 5bd, 7 (If only you had a side of the right angle triangle…), 9 (what is an angle of depression??), 10, 13