1.5 Evaluating Limits

It will be helpful for you to remember that a **Limit of a Function** is a **potential functional value**. Because functional values are **simply numbers**, then **limits are just numbers** too. And so, to **evaluate** a limit **means** to **calculate the number that the limit is**.

On Page 40 of your text we see seven Properties of Limits listed. These seven properties can be thought of as the algebra of limits (where we think of "algebra" as a "set of rules for calculating").

The first two properties are important enough that we should look at them in some small detail.

1) $\lim_{x \to a} (k) =$

2) $\lim_{x \to a} (x) =$

The five other properties allow us to "use" the above two.

We will see that **using the Properties of Limits is a mathematical practice** very much like what you've been doing with algebra over the last few years. The **context** is just a little different. Instead of solving an equation, we are calculating potential functional values.

Example 1.5.1

Using the Properties of Limits, determine $\lim_{x \to 2}\left(\frac{3x^2 - 5x}{x+3}\right)$.

To be frank, it seems a little silly and certainly is tedious to evaluate limits by stating the various properties as we use them. In fact, using the various Properties of Limits can be boiled down to a single statement (or single Limit Law, if you will allow):

One of three things will happen:

1) You calculate a definite (a finite) number.

Example 1.5.2

Evaluate $\lim_{x \to -3} \left(\frac{2x-5}{7x} \right)$

2) Your calculation arrives at $\frac{\text{definite non-zero \#}}{0}$

Example 1.5.3

Evaluate $\lim_{x \to 2} \left(\frac{3x^2-5}{x-2} \right)$

3) Your calculation arrives at an **indeterminate** form:

$$\frac{0}{0}, \text{ or } \frac{\infty}{\infty}, \text{ or } 0 \cdot \infty$$

Example 1.5.4

Determine the limits, if they exist:

a) $\lim_{x \to 1} \left(\frac{2x^2 + x - 3}{x - 1} \right)$

b) $\lim_{x \to 3} \left(\frac{\sqrt{x^2 - 5} - 2}{x - 3} \right)$

c) $\lim_{x \to 0}\left(\frac{(x+27)^{\frac{1}{3}}-3}{x}\right)$

Class/Homework for Section 1.5

*Pg. 45 – 47 #3, 4, 7, 8abc (for #3, **explain** your thinking)*