1.6 Continuity

Before embarking on the wonder filled road that is "Continuity", we should take another quick look at a couple of examples in Limit Evaluation (Section 1.5). Before looking at the examples, however, let's consider the definition of the Absolute Value.

Definition 1.5.1

The Absolute Value of x, written $|x|$ is defined as:

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Example 1.5.5

Determine the limit, if it exists:

$$\lim_{x \to \frac{5}{2}} \left(\frac{2x-5}{|2x-5|} \right)$$

Example 1.5.6

Determine the limit, if it exists:

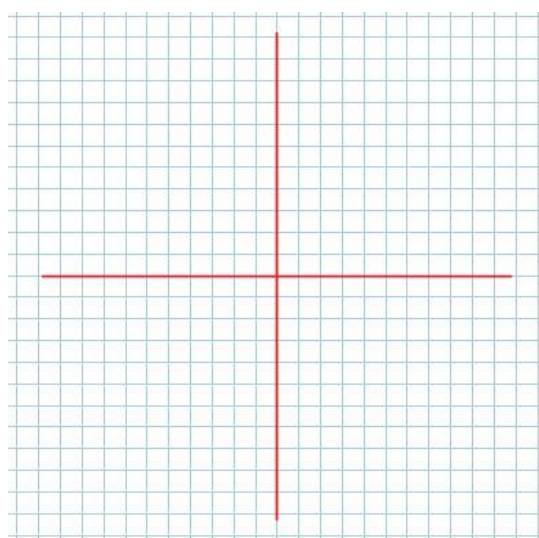
$$\lim_{x\to 1}\left(\sqrt{x-1}\right)$$

And now on to Continuity

A Geometric View

A function, $f(x)$, is continuous (cts) if its sketch can be drawn without lifting your pen/pencil from the page.

Example 1.6.1



An Algebraic Definition (*Memorize!*)

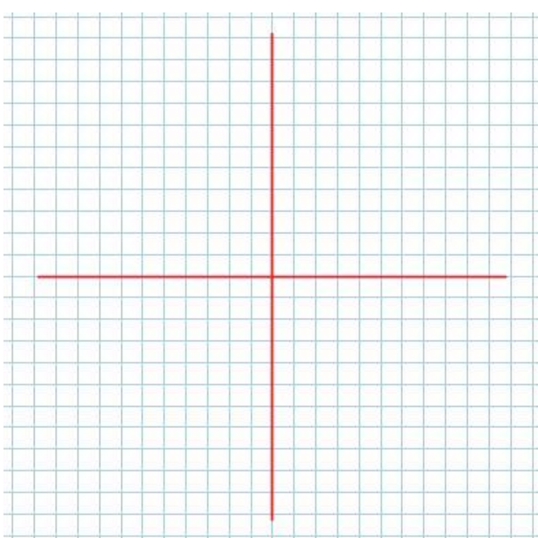
Definition 1.6.1

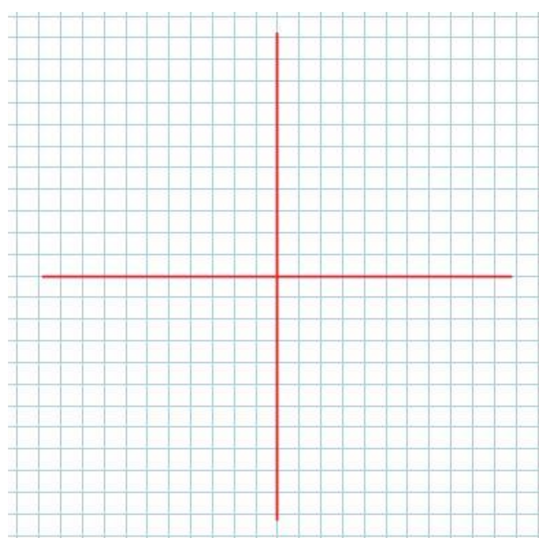
A function $f(x)$ is **continuous** at (the domain value) $x = a$ if:

Note: If **any** of these conditions is/are **not met**, we say the function is **discontinuous** at

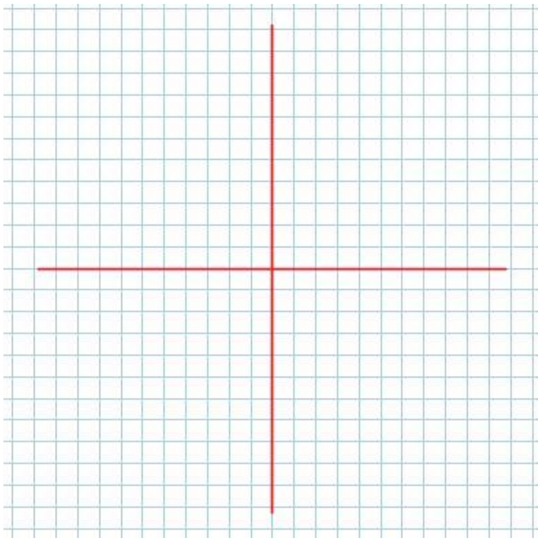
Recall the three types of discontinuities:

1) Hole

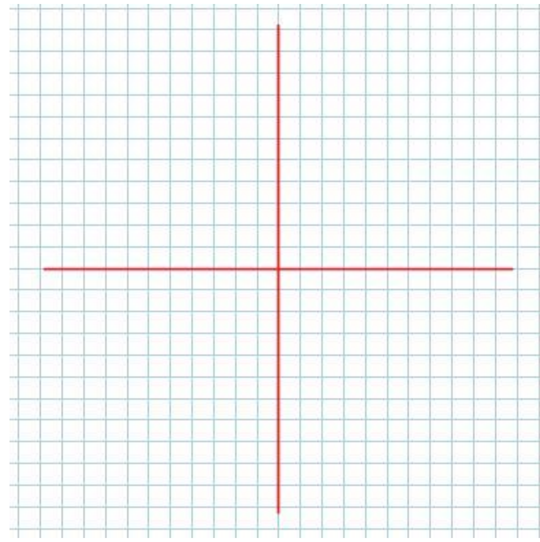




2) Jump



3) Infinite (or Asymptotic)



Continuity at a **single point** is vital for Differential (and Integral) Calculus, **BUT** functions are defined over

We CANNOT check to determine whether a function is continuous (or not) over

Thankfully we have the following results:

1) Polynomial Functions

2) Rational Functions (*recall the definition of a rational function*)

$$\left(R(x) = \frac{P(x)}{Q(x)},\ Q(x) \neq 0,\ P(x) \text{ and } Q(x) \text{ both polynomials} \right)$$

3) Radical Functions

4) Exponential, Logarithmic and Trigonometric Functions

5) Piecewise Defined Functions

Example 1.6.2

Determine where the function is continuous:

$$f(x) = \begin{cases} 3x^2 - 1, & x \geq 0 \\ x - 1, & x < 0 \end{cases}$$

Example 1.6.3

Determine if $g(x)$ is cts at $x=3$:

$$g(x)=\begin{cases}\dfrac{2x^2-5x-3}{x-3}, & x\neq 3\\ 6, & x=3\end{cases}$$

Class/Homework for Section 1.6

Pg. 52 – 53 #3 – 5,7 – 8, 10, 12 – 15