A∞Ω
MATH@TD

MCV4U Practice for the Chapter 1 Quiz

Multiple Choice

Identify the choice that best completes the statement or answers the question.

____ 1. Determine the slope of the line through the points (8, 13) and (–2, 3).

a. 1
b. –1
c. $\sqrt{10}$
d. $-\sqrt{10}$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-13}{-2-8} = +1 \quad a$$

2. Determine an expression, in simplified form, for the slope of the secant PQ with $P(1, 2)$ and $Q(1 + h, f(1+h))$ where $f(x) = 2x^2$.

a. $4+2h$
b. $4-2h$
c. $8-4h$
d. $8+4h$

$$m_{sec} = \frac{f(1+h) - f(1)}{(1+h) - 1} = \frac{2(1+h)^2 - 2(1)^2}{h}$$
$$= \frac{2(1+2h+h^2) - 2}{h}$$
$$= \frac{4h+2h^2}{h} = 4+2h \quad a$$

3. Determine an equation of the line tangent to the curve $y = \frac{1}{x+3}$ at the point with x-coordinate 2.

a. $-7x+25y+1=0$
b. $7x-25y+1=0$
c. $-x-25y-7=0$
d. $x+25y-7=0$

Need a point "(x,y)" and a slope

POINT
when $x=2$
$y=\frac{1}{5}$
$\Rightarrow (2, \frac{1}{5})$

SLOPE
$$m_{tan} = \lim_{h\to 0}\left(\frac{\frac{1}{(2+h)+3} - \frac{1}{2+3}}{h}\right) = \lim_{h\to 0}\left(\frac{\frac{1}{h+5} - \frac{1}{5}}{h}\right)$$
$$= \lim_{h\to 0}\left(\frac{5-(h+5)}{h(5)(h+5)}\right)$$
$$= \lim_{h\to 0}\left(\frac{-\cancel{h}}{\cancel{h}(5)(h+5)}\right)$$
$$= -\frac{1}{25}$$

$\therefore$ (point-slope form)
eqn of line is
$$y - y_1 = m(x - x_1)$$
$$\Rightarrow y - \frac{1}{5} = -\frac{1}{25}(x-2)$$
$\times 25$
$$\Rightarrow 25y - 5 = -x + 2$$
$$\Rightarrow \boxed{x + 25y - 7 = 0} \quad d$$

standard form.

4. An oil tank is being drained for cleaning. After t minutes there are V litres of oil left in the tank, where $V(t) = 40(20-t)^2$, $0 \le t \le 20$. Determine the rate of change of volume at the time $t = 10$.

a. -800 litres/minute
b. -600 litres/minute
c. -400 litres/minute
d. -200 litres/minute

instantaneous rate of change

$$IROC = \lim_{h\to 0}\left(\frac{V(10+h) - V(10)}{h}\right) = \lim_{h\to 0}\left(\frac{40(20-(10+h))^2 - 40(20-10)^2}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{40(10-h)^2 - 40(10)^2}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{40(100-20h+h^2) - 40(100)}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{-800h + 40h^2}{h}\right) = \lim_{h\to 0}\left(\frac{\cancel{h}(-800+40h)}{\cancel{h}}\right) = -800 \text{ l/min} \quad a$$

5. What is the slope of the tangent to the graph of the position function?

a. instantaneous position
b. instantaneous acceleration
c. average velocity
d. instantaneous velocity

" $\frac{\text{position}}{\text{time}}$ " is velocity $\Rightarrow$ slope of tangent would be instantaneous velocity d

6. Determine $\lim_{x\to -3} \frac{2x^3 - 18x}{x+3}$.

a. 6
b. -6
c. 36
d. -36

$$\lim_{x\to -3}\left(\frac{2x^3 - 18x}{x+3}\right) \quad \text{" } \frac{0}{0} \text{ "}$$

$$= \lim_{x\to -3}\left(\frac{2x(x^2-9)}{x+3}\right)$$

$$= \lim_{x\to -3}\left(\frac{2x(x-3)\cancel{(x+3)}}{\cancel{x+3}}\right)$$

$$= 2(-3)(-3-3) = (-6)(-6) = +36. \quad c$$

7. $d(x) = \begin{cases} -x-k, \text{ if } x \neq -1 \\ 2x+2k, \text{ if } x = -1 \end{cases}$. Determine k so that $d(x)$ is continuous.

a. $k = 1$ c. $k = 0$

b. $k = -1$ d. $k = -4$

The 'problem' occurs at $x = 1$

we need $\lim_{x \to 1} (d(x)) = d(1)$

$$\Rightarrow \lim_{x \to 1} (-x-k) = 2(1) + 2k$$

$$\Rightarrow -1-k = 2+2k$$

$$\Rightarrow 3k = -3 \Rightarrow k = -1 \quad \text{(b)}$$

8. Rationalize the denominator of $\frac{\sqrt{10}}{3\sqrt{3}+\sqrt{15}}$.

$$= \frac{\sqrt{10}}{3\sqrt{3}+\sqrt{15}} \cdot \frac{3\sqrt{3}-\sqrt{15}}{3\sqrt{3}-\sqrt{15}}$$

$$= \frac{(\sqrt{10})(3\sqrt{3}-\sqrt{15})}{9(3)-(15)}$$

$$= \frac{3\sqrt{30}-\sqrt{150}}{27-15}$$

→ numerator could be left like this

$$\Rightarrow = \frac{3\sqrt{30}-\sqrt{25\cdot 6}}{12}$$

$$= \frac{3\sqrt{30}-5\sqrt{6}}{12}.$$

nicer though.

9. Determine an equation of the line tangent to the curve $y = \sqrt{5x-4}$ at the point with x-coordinate 4.

need point and slope

point: when $x = 4$
$\rightarrow y = 4$
$\rightarrow (4,4)$

slope: $m_{tan} = \lim_{h \to 0} \left(\frac{\sqrt{5(4+h)-4} - \sqrt{5(4)-4}}{h} \right)$

$$= \lim_{h \to 0} \left(\frac{\sqrt{16+5h} - \sqrt{16}}{h} \right)$$

$$= \lim_{h \to 0} \left(\frac{\sqrt{16+5h} - 4}{h} \cdot \frac{\sqrt{16+5h}+4}{\sqrt{16+5h}+4} \right)$$

$$= \lim_{h \to 0} \left(\frac{16+5h-16}{h(\sqrt{16+5h}+4)} \right)$$

$$= \lim_{h \to 0} \left(\frac{5\cancel{h}}{\cancel{h}(\sqrt{16+5h}+4)} \right)$$

$$= \frac{5}{8}$$

$\boxed{\text{slope-point}}$ ∴ eqn is

$$y - 4 = \frac{5}{8}(x-4)$$

← acceptable

→ STD FORM

$$5x - 8y - 12 = 0$$

10. Describe what can be inferred about the line tangent to a curve if the slope at a point is found to be 0.

The tangent line is horizontal.

11. Determine the average velocity of the function $f(t) = \sqrt{t-2}$ between the time intervals $t = 3$ and $t = 5$.

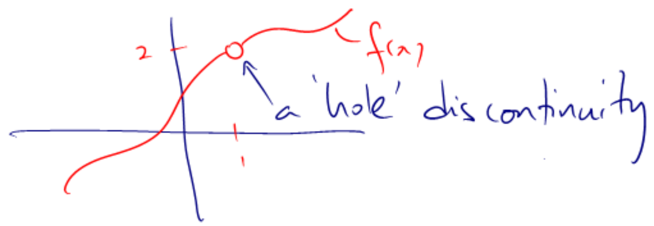
$$V_{avg} = \frac{f(5) - f(3)}{5-3} = \frac{\sqrt{5-2} - \sqrt{3-2}}{2} = \frac{\sqrt{3}-1}{2}.$$

12. Does the value of a function at a point have to exist in order for the limit to exist at that point? Explain.

No. A picture is worth a thousand words:

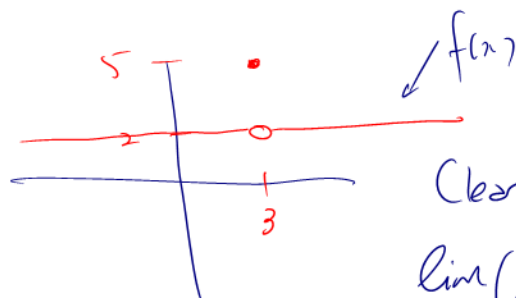
Clearly $f(1)$ dne.

BUT $\lim\limits_{x \to 1}(f(x)) = 2$.



13. Does $\lim\limits_{x \to 3} \begin{cases} 5, & \text{if } x = 3 \\ 2, & \text{if } x \neq 3 \end{cases}$ exist? Explain.

Yes. Picture:



Clearly $\lim\limits_{x \to 3}(f(x)) = 2$

14. Determine $\lim\limits_{x \to 3} \dfrac{3x-8}{4x-12}$, if it exists.

$$\lim_{x \to 3}\left(\frac{3x-8}{4x-12}\right) = \frac{1}{0} \quad \therefore \text{dne.}$$

15. Determine $\lim\limits_{x \to -9} \dfrac{5x^3 + 40x^2 - 45x}{x+9}$.

$$= \lim_{x \to -9}\left(\frac{5x(x^2+8x-9)}{(x+9)}\right) \quad \text{"}\frac{0}{0}\text{"}$$

$$= \lim_{x \to 9}\left(\frac{5x(x\cancel{+9})(x-1)}{x\cancel{+9}}\right)$$

$$= 5(9)(8)$$

$$= 360$$

16. Explain how to determine $\lim_{x \to 3} \frac{2x-6}{x^2-9}$.

① "plug in" $x = 3 \Rightarrow$ gives "$\frac{0}{0}$"

② factor numerator and denominator

$$\lim_{x \to 3} \left(\frac{2(x-3)}{(x-3)(x+3)} \right)$$

③ cancel common factors "$(x-3)$"

④ plug in 3 again $\Rightarrow \frac{2}{6} = \frac{1}{3}$.

17. $j(x) = \begin{cases} x-2, \text{ if } x \neq -2 \\ 3kx+5, \text{ if } x = -2 \end{cases}$. Determine k so that $j(x)$ is continuous.

We need

$$\lim_{x \to -2} (j(x)) = j(-2)$$

$$\Rightarrow \lim_{x \to -2} (x-2) = 3k(-2) + 5$$

$$-4 = -6k + 5$$

$$6k = 9 \Rightarrow k = 3/2$$

18. Determine the values of x for which the function $f(x) = \frac{\sqrt{3x-6}}{x-5}$ is continuous.

$f(x)$ is continuous everywhere on its domain

The numerator requires $3x - 6 \geq 0$ (can't have negatives in square roots)

$\Rightarrow x \geq 2$

But the denominator gives a vertical asymptote at $x = 5$

$\therefore f(x)$ is continuous on $[2, 5) \cup (5, \infty)$

(or $\{x \in \mathbb{R} \mid x \geq 2, x \neq 5\}$.)