Chapter 1 – Introduction to Calculus - ***Review***

In Chapter 1 we learned some of the **fundamental** concepts of (differential) Calculus. Two of the concepts we considered are **key** to your understanding Calculus, so make sure you understand the meaning of the *Limit of a Function*, and *Continuity of Functions*. We also reviewed how to rationalize expressions with radicals, and saw (finally) how to calculate the Instantaneous Rate of Change (or the slope of a tangent) of a function at a point.

Section 1.1 – Rationalizing

Keep in mind the notion of a conjugate: e.g. Given $a + b$ we call $a - b$ the conjugate.

Section 1.2 – The Slope of a Tangent (or The Instantaneous Rate of Change)

With a small modification to the **difference quotient**, we defined the **slope of a tangent** to a function, $f(x)$ at a domain value $x = a$ to be:

$$m_{\tan} = \lim_{h \to 0}\left(\frac{f(a+h) - f(a)}{h}\right)$$

You must have this definition memorized, and you must know how to use it.

Section 1.4 – The Limit of a Function

In this section we considered the concept of a limit, and learned that a **limit can be thought of** as a "**potential functional value**". The main point of considering limits is to be able to explore the behavior of a function near any problem domain values. We learned two definitions:

Definition:

We take $\lim_{x \to a}\left(f(x)\right) = L$ to mean that as x gets "ridiculously" close to the domain value a, the functional value is getting "ridiculously" close to the finite number L.

Definition:

Given the function $f(x)$ then $\lim_{x \to a}\left(f(x)\right) = L$ exists **if and only if**

$$\lim_{x \to a^-}\left(f(x)\right) = L = \lim_{x \to a^+}\left(f(x)\right)$$

Section 1.5 – Evaluating Limits

Here we saw the "**seven properties of limits**" and that they actually boil down to the statement "**Plug it in and see what happens**". When evaluating limits, there are three possible outcomes at the "plug it in…" stage. Those possibilities are:

- We obtain a definite answer, and so the problem is done.
- We obtain an answer which is undefined (i.e. something like $\frac{2}{0}$) and again, the problem is complete.
- We obtain an **indeterminate** form $\frac{0}{0}$, $\frac{\infty}{\infty}$, or $0 \cdot \infty$ and so "**more work**" must be done to **determine** an answer.

Section 1.6 – Continuity

In this final section of the chapter we learned the definition of continuity of a function at a point. You must have the definition memorized.

Definition:

A function, $f(x)$, is continuous at the domain value $x = a$ if:

1. $f(a)$ is defined.
2. $\lim_{x \to a}(f(x))$ exists.
3. $\lim_{x \to a}(f(x)) = f(a)$

We also learned that all of the functions we will consider in the Calculus half of the course are "nicely behaved". That is, **the functions we consider are all continuous on their domains**, **with the exception of piecewise defined functions which will require some analysis to determine their continuity.** Make sure you **know the three types of discontinuities**!

Some examples:

1. Rationalize the numerator

$$\frac{\sqrt{x-2}+3}{x-7}$$

2. Determine the **equation** of the tangent to the function:

a) $f(x) = 2x^3 + 3x - 1$ at $x = 0$

b) $g(x) = 2\sqrt{x+1} - 4$ at $x = 3$

3. Evaluate the following limits:

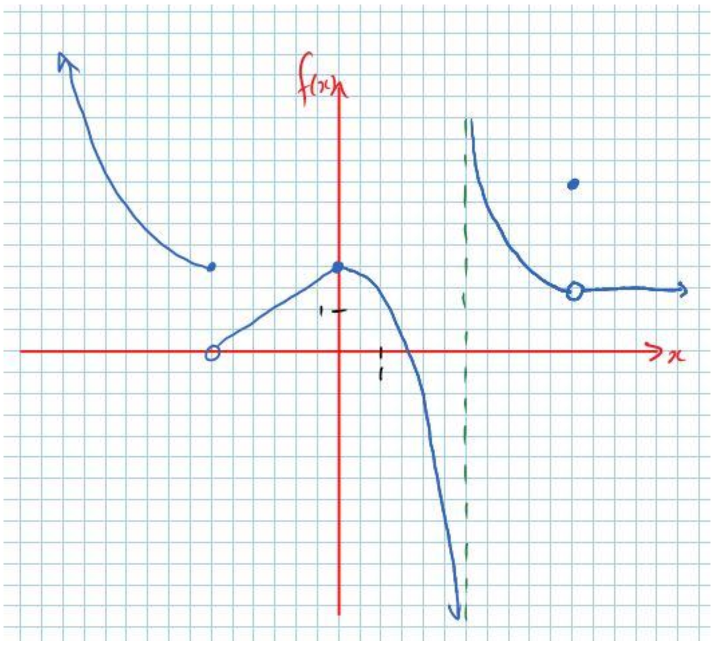
a) $\lim_{x\to 5}\left(\frac{x^2-25}{(x-5)^2}\right)$

b) $\lim_{x\to 2}\left(\frac{3x+5}{x-3}\right)$

c) $\lim_{t\to 6}\left(\frac{\sqrt{2t-3}-3}{t^2-36}\right)$

d) $\lim_{x\to -1}\left(\frac{\sqrt[3]{x}+1}{x+1}\right)$

4. The following sketch of the graph of the function $f(x)$ shows three discontinuities. State where the discontinuity is, what the type of discontinuity it is, and which condition of continuity is being broken.



5. Show that the given function is continuous for all real numbers.

$$f(x) = \begin{cases} 2x^2 + 1, & x \le 0 \\ \cos(x), & x > 0 \end{cases}$$