MCV4U

Homework Help

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#21 $f(x) = 3x^2 - 4x$ Tangent parallel to $y = 8x$ $\quad m = 8$

$m_{tan} = 8$

$$m_{tan} = \lim_{h \to 0} \left(\frac{f(x+h) - f(x)}{h} \right)$$

$$= \lim_{h \to 0} \left(\frac{(3(x+h)^2 - 4(x+h)) - (3x^2 - 4x)}{h} \right)$$

$$= \lim_{h \to 0} \left(\frac{3(x^2 + 2xh + h^2) - 4x - 4h - 3x^2 + 4x}{h} \right)$$

$$= \lim_{h \to 0} \left(\frac{6xh + 3h^2 - 4h}{h} \right)$$

$$= \lim_{h \to 0} \left(\frac{h(6x + 3h - 4)}{h} \right)$$

$$= \lim_{h \to 0} (6x + 3(0) - 4)$$

$$= 6x - 4$$

Since $m_{tan} = 6x - 4$

$\therefore$ we want $m_{tan} = 8$

$\Rightarrow$ set $6x - 4 = 8$

$\Rightarrow 6x = 12$

$\therefore x = 2$

$f(2) = 3(2)^2 - 4(2)$

$= 12 - 8$

$= 4$

$\therefore$ The point is $(2, 4)$

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#22 $y = \frac{1}{3}x^3 - 5x - \frac{4}{x}$ Horizontal Tangent - find x.

$m_{tan} = 0$

$$m_{tan} = \lim_{h\to 0}\left(\frac{\left[\frac{1}{3}(x+h)^3 - 5(x+h) - \frac{4}{x+h}\right] - \left[\frac{1}{3}x^3 - 5x - \frac{4}{x}\right]}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{\frac{1}{3}\left(x^3 + 3x^2h + 3xh^2 + h^3\right) - 5x - 5h - \frac{4}{x+h} - \frac{1}{3}x^3 + 5x + \frac{4}{x}}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{x^2h + xh^2 + \frac{1}{3}h^3 - 5h - \frac{4}{x+h} + \frac{4}{x}}{h}\right)$$

"C.D." $3(x+h)x$

$$= \lim_{h\to 0}\left(\frac{x^2h(3(x+h)(x)) + xh^2(3(x+h)(x)) + h^3(x+h)(x) - 5h(3(x+h)x) - 4(3x) + 4(3(x+h)(x))}{3(x+h)(x)}\Big/\frac{h}{1}\right)$$

$$= \lim_{h\to 0}\left(\frac{h\left[3x^3(x+h) + 3x^2h(x+h) + h^2x(x+h) - 15x(x+h) + 12\right]}{h\left(3x(x+h)\right)}\right)$$

$$\Rightarrow m_{tan} = \left(\frac{3x^4 - 15x^2 + 12}{3x^2}\right) \quad \text{we want} \quad m_{tan} = 0$$

$$\Rightarrow \frac{3x^4 - 15x^2 + 12}{3x^2} = 0$$

$\div 3 \Rightarrow 3x^4 - 15x^2 + 12 = 0$

$\Rightarrow x^4 - 5x^2 + 4 = 0 \quad | \; x^2 - 5x + 4 \Rightarrow$

$\Rightarrow (x^2-4)(x^2-1) = 0 \quad | \; (x-4)(x-1) = 0$

$\Rightarrow x^2 - 4 = 0$ or $x^2 - 1 = 0$

$(x-2)(x+2) = 0 \qquad (x-1)(x+1) = 0$

$\Rightarrow x = 2$ or -2 or $x = 1$ or $x = -1$