7.6 The Cross Product

The Cross Product is sometimes called the Vector Product because it produces a vector.

Note: 1) The cross product is only used in R³.
2) Given two vectors *a* and *b* in R³ the cross product

Picture:

3) We determine the **direction** of

4) Clearly

Algebraic View of the Cross Product

Consider: $\hat{i} \times \hat{j} =$



Consider two general vectors in \mathbb{R}^3 : $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$. If we write the vectors as linear combinations of the standard unit vectors we can use the above ideas to calculate $\vec{a} \times \vec{b}$.

Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$. Then:

$$\vec{a} \times \vec{b} =$$

Another "pattern" for developing the algebraic cross product

Example 7.6.1

Given $\vec{a} = (3, -2, 5)$ and $\vec{b} = (2, 1, 0)$ determine $\vec{a} \times \vec{b}$.

Example 7.6.2 Given $\vec{a} = (3,1,0)$ and $\vec{b} = (4,-1,3)$, find: a) $\vec{a} \cdot (\vec{a} \times \vec{b})$ b) $\vec{b} \cdot \left(\vec{a} \times \vec{b}\right)$

c)
$$\vec{a} \times (\vec{a} \cdot \vec{b})$$

Q. Of the following two vector expressions, which is meaningful? i) $(\vec{a} \cdot \vec{b})(\vec{a} \times \vec{b})$, or ii) $\vec{a} \times (\vec{a} \times \vec{b})$

Class/Homework for Section 7.6 Pg. 405 Investigation (optional) Pg. 407 – 408 #1, 3, 4def, 5 – 7, 8a, 11