7.7 Applications of the Dot and Cross Products

Given two non collinear vectors $\vec{a},\vec{b} \in \mathbb{R}^3$, with $\vec{a} = (a_1, a_2, a_3)$, and $\vec{b} = (b_1, b_2, b_3)$ then:

Dot Product

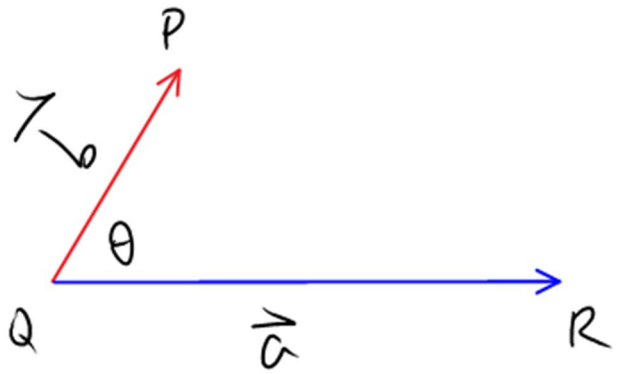
Geometric Form	Algebraic Form

Cross Product

Geometric Form	Algebraic Form

Consider the fact that $\left|\vec{a} \times \vec{b}\right| = \left|\vec{a}\right|\left|\vec{b}\right| \sin(\theta)$ (see Pg. 411 for a proof). Further consider a **unit vector** $\hat{n}$ perpendicular to the plane containing $\vec{a}$ and $\vec{b}$. Then the geometric form of $\vec{a} \times \vec{b}$ is

Consider the following picture (note: $\vec{a}, \vec{b} \in \mathbb{R}^3$) :



The parallelogram arising from the vectors $\vec{a}$ and $\vec{b}$ has an area given by:

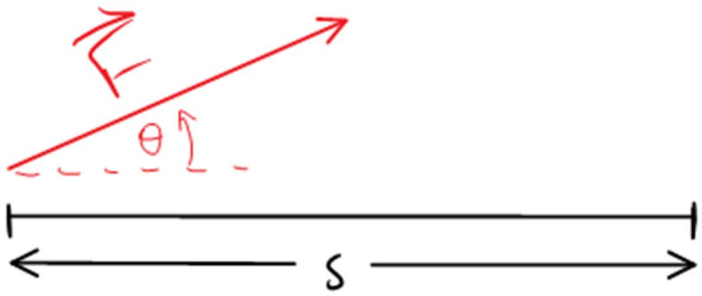
Example 7.7.1

Calculate the area of a triangle with vertices $A(2,0,1)$, $B(1,1,2)$ and $C(4,1,3)$.

Two Applications

Dot Product and Work

Physics tells us that $Work = \left(Force\ applied\right)\left(displacement\right)$. Consider the picture:



$\left|F_{app}\right| =$

Thus

$W =$

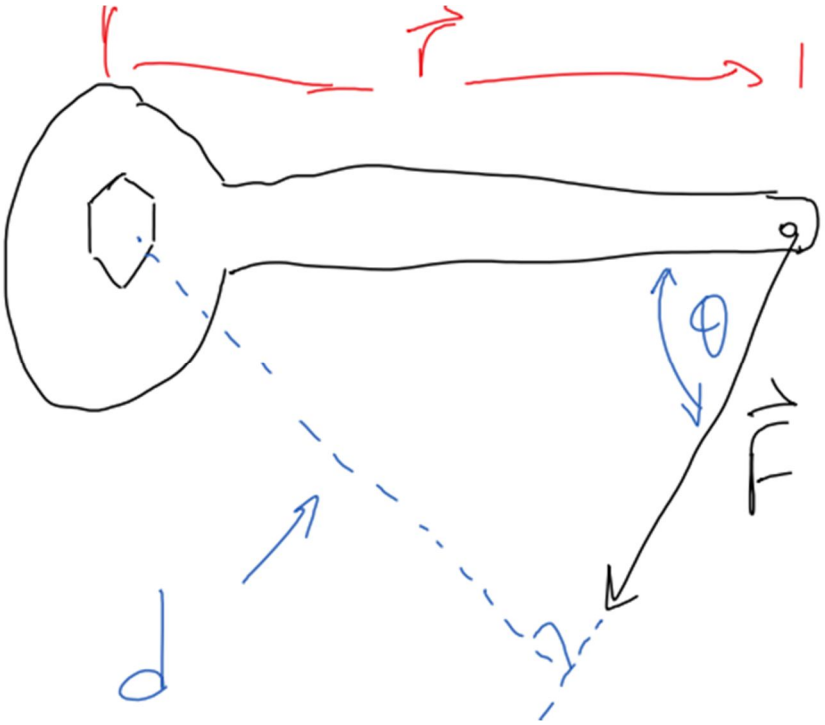
Example 7.7.2

From your text: Pg. 415 #3b

Calculate the amount of work done when a $40kg$ rock falls $40m$ down a slope at 50° to the vertical.

Cross Product and Torque

Torque is the "twisting" force around a turning point. (e.g. the force on a bolt by a wrench.) Consider the (poorly drawn) picture:



Torque is the force applied at the **bolt**.

Q. How can we **maximize** torque?

Class/Homework for Section 7.7

Pg. 415 #2, 3, 5, 6, 8, 10