

## Vectors Half (Chapters 6 – 9) – *Exam Overview*

Congratulations! You have made it through Calculus and Vectors (final exam still to come!) Below is a rough overview of what you are expected to know. Study well! **Read your notes**, and the **text examples** (and the “**In Summary**” **stuff at the end of the chapter sections**).

## Geometry of Vectors

1. Know the basic characteristics of a vector.
2. Be able to add/subtract vectors “geometrically” using the cosine law.

## Algebra of Vectors (Note: There are close ties to Geometry with much of these topics)

1. Given 4 (algebraic – i.e. in component form) vectors  $\vec{a}, \vec{b}, \vec{c}$  and  $\vec{d}$  in  $\mathbb{R}^2$  or  $\mathbb{R}^3$ , determine:
  1. Linear combinations through add/subt/scalar multiplication (be able to use all).
  2. Magnitude of  $\vec{a}$
  3. Unit vector in the direction of  $\vec{b}$
  4. Angle between  $\vec{c}$  and  $\vec{d}$ .
  5. Dot Product of two vectors.
  6. Cross Product of two vectors in  $\mathbb{R}^3$ . Why does cross product not work in  $\mathbb{R}^2$ ?
  7. Scalar Projections ( $\text{comp}_{\vec{a}} \vec{b}$ ) and Vector Projections ( $\text{proj}_{\vec{a}} \vec{b}$ )
2. Be able to write vectors as Linear combinations of the Standard Unit Vectors  $\hat{i}$ ,  $\hat{j}$ , and  $\hat{k}$ .
3. Force is a valid option for an application question.
4. Work is a valid option for an applications question.
5. Torque is a valid option for an application question.
6. Know your Direction Angles and Direction Cosines.
7. Vector or scalar?
  1. What is the difference?
  2. One is nonsense, one is a vector, one is a scalar. Determine which is which...
    1.  $(\vec{a} \cdot \vec{b}) \times \vec{c}$
    2.  $\vec{a} \cdot (\vec{b} \times \vec{c})$
    3.  $(\vec{a} \cdot \vec{b})(\vec{b} \times \vec{c})$
8. Span of a Set of Vectors.

## Lines

1. Given the correct info in  $\mathbb{R}^2$  (two points, or a direction vector and a known point), determine vector, parametric, symmetric, and scalar equations of a line.
  1. State a normal to the line. State a direction vector of the line. Use dot product to prove these vectors are perpendicular.
  2. Determine the distance from a given point to the line, or between parallel lines.
2. Determine all equations of a line in  $\mathbb{R}^3$ . Why are there no “scalar” equations of lines in  $\mathbb{R}^3$ ?
3. Distance in  $\mathbb{R}^3$  as well!
4. What are skew lines?
5. The angle between intersecting lines is the same as the angle between their direction vectors.

## Planes

1. Given points  $A$ ,  $B$ ,  $C$ , and  $D$ , (or two direction vectors, or a direction vector and enough info to get a second direction vector) determine:
  1. The Vector, Parametric and Cartesian equations of the plane through the points  $A$ ,  $B$ ,  $C$ .
  2. Why are there no symmetric equations of planes?
  3. The Vector equation of the line through a point  $D$  perpendicular to the plane of the first part.
  4. The distance from  $D$  to the plane of the first part.
  5. Distance between parallel planes.
  6. The angle between planes is the same as the angle between their normals.
  7. The angle between a line and a plane is  $90^\circ - \theta$ , where  $\theta$  is the angle between the normal of the plane and the direction of the line.
2. Determine the solutions, if any, the following.
  1. Intersecting Lines with Lines and Lines with Planes (you may need to “parameterize” your solution set).
  2. Systems of Equations (feel free to use Gaussian Elimination or Gauss-Jordan Elimination).

## Suggested Review Problems

Chapter 6 (**Vector Basics**): Pg. 344 – 347 #4 – 8, 11, 16, 17, 18, 21

Chapter 7 (**Some Applications of Vectors**): Pg. 418 – 421 #2, 4, 7, 9, 10b, 12, 14, 18, 20, 22, 30

Chapter 8 (**Equations of Lines and Planes**): Pg. 480 – 483 #3 – 6, 8, 9, 11, 20, 24, 29, 34

Chapter 9 (**Intersections and Systems of Equations**): Pg. 552 – 555 #5, 6, 12a (hint:  $\vec{m} \cdot \vec{n}$ ), 13a, 14, 19

Cumulative Review: Pg. 557 – 560 #1, 3 – 7, 11, 18, 20, 21, 22, 25ab, 28, 31, 34