1.4b The Limit of a Function (Con't)

Example 1.4b.1

Determine $\lim_{x \to 3}\left(f(x)\right)$ for

$$f(x) = \begin{cases} x-4, & x \le 3 \\ x^2 - 10, & x > 3 \end{cases}$$

behavior for $x<3$

behavior for $x>3$ $(x \to 3^+)$

$\therefore$ the final behavior changes at $x=3$ $\Rightarrow$ TWO ONE SIDED LIMITS.

$$\lim_{x \to 3^-}\left(f(x)\right) = \lim_{x \to 3^-}(x-4) = -1$$

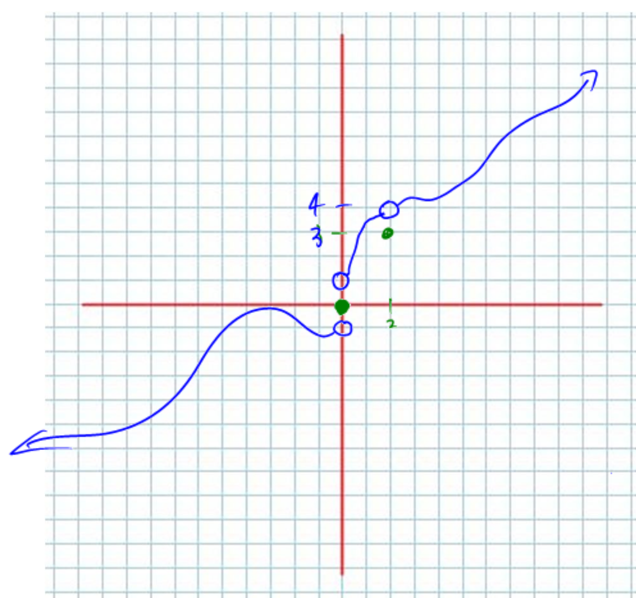
$$\lim_{x \to 3^+}\left(f(x)\right) = \lim_{x \to 3^+}(x^2-10) = -1$$

$$\therefore \lim_{x \to 3}\left(f(x)\right) = -1$$

Example 1.4b.2

Sketch the graph of a function $f(x)$ with the following properties:

1) $\lim_{x \to 0^-}\left(f(x)\right) = -1$
2) $\lim_{x \to 0^+}\left(f(x)\right) = +1$
3) $f(0) = 0$
4) $\lim_{x \to 2}\left(f(x)\right) = 4$
5) $f(2) = 3$



Class/Homework for Section 1.4b

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$(8+h)(8+h) = 64 + 8h + 8h + h^2$

1. Simplify the Difference Quotients:

a. $\frac{(8+h)^2 - 64}{h}$ $= \frac{64 + 16h + h^2 - 64}{h}$

$= \frac{16h + h^2}{h} = \frac{\cancel{h}(16+h)}{\cancel{h}}$, $h \neq 0$

$= 16 + h$

2. Determine, and simplify, an expression describing the slope of a secant through the given points:

b. $A(1, f(1))$, $B(1+h, f(1+h))$, where $f(x) = 2x^3 - 1$

c. $R(0,2)$, $S\left(h, \sqrt{h+4}\right)$

b) $m = \frac{f(1+h) - f(1)}{h} = \frac{(2(1+h)^3 - 1) - (2(1)^3 - 1)}{h}$

$= \frac{2(1 + 3h + 3h^2 + h^3) - 1 - 1}{h}$

$= \frac{6h + 6h^2 + 2h^3}{h} = \frac{\cancel{h}(6 + 6h + 2h^2)}{\cancel{h}}$, $h \neq 0$

$= 6 + 6h + 2h^2$

$R(0,2)$, $S(h, \sqrt{h+4})$

If you have a radical you likely need to conjugate.

$$m_{sec} = \frac{\sqrt{h+4} - 2}{h} \cdot \frac{\sqrt{h+4} + 2}{\sqrt{h+4} + 2}$$

$$= \frac{h+4-4}{h(\sqrt{h+4}+2)} = \frac{\cancel{h}}{\cancel{h}(\sqrt{h+4}+2)} \quad , h \neq 0$$

$$= \frac{1}{\sqrt{h+4}+2}$$