Hwk Check

Q 1.4b

5. Determine the real values of a, b, and c for the quadratic function $f(x) = ax^2 + bx + c,\ a \neq 0$, that satisfy the conditions:
$f(0) = 0,\ \lim_{x\to 1}(f(x)) = 5$, and $\lim_{x\to -2}(f(x)) = 8$.

$f(0) = 0$

$0 = a(0)^2 + b(0) + c$

$\Rightarrow c = 0$

$\lim_{x\to 1}(ax^2 + bx) = 5$

$\Rightarrow a + b = 5$ ①

$\lim_{x\to -2}(ax^2 + bx) = 8$

$\Rightarrow 4a - 2b = 8$ $\div 2$

$2a - b = 4$ ②

① & ② form a System of Linear Eqns (SOLE)

$a + b = 5$ ①

$2a - b = 4$ ②

① + ② $3a = 9$

$a = 3$ substitute $a = 3$ into ①

$3 + b = 5 \Rightarrow b = 2$

$\therefore a = 3,\ b = 2,\ c = 0$ ✓

§ 1.4b

3. Sketch the graph of each piecewise defined function and determine the indicated limit. If the limit does not exist, explain the problem:

   a. $f(x)=\begin{cases} x+1, & x<0 \\ \cos(x), & x \geq 0 \end{cases}$; $\lim\limits_{x\to 0}(f(x))$

$\lim\limits_{x\to 0^-}(f(x))$

$= \lim\limits_{x\to 0^-}(x+1)$

$= 1$

$\lim\limits_{x\to 0^+}(f(x))$

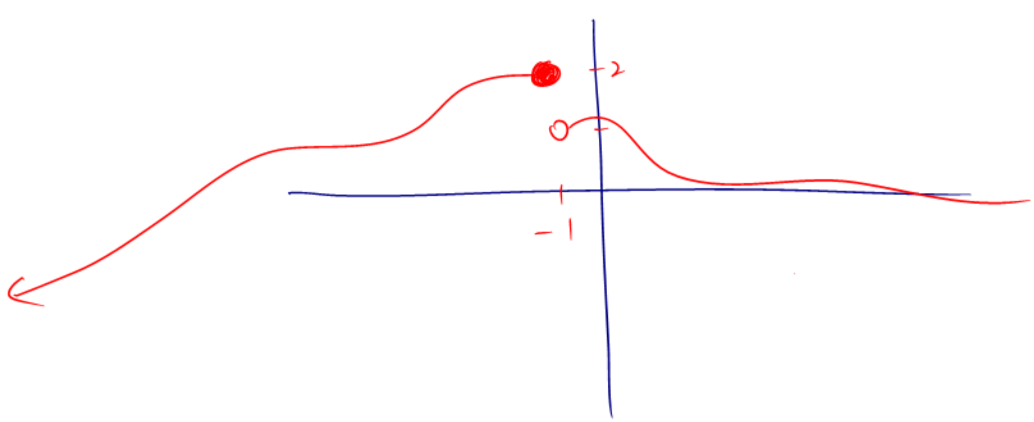
$= \lim\limits_{x\to 0^+}(\cos(x))$

$= 1$

$\therefore \lim\limits_{x\to 0}(f(x)) = 1$

4. Sketch the possible graph of a function with the given characteristics:

   $\lim\limits_{x\to -1^-}(f(x))=2,\ \lim\limits_{x\to -1^+}(f(x))=1,\ f(-1)=2$



1.5 Evaluating Limits

It will be helpful for you to remember that a **Limit of a Function** is a **potential functional value**. Because functional values are **simply numbers**, then **limits are just numbers** too. And so, to **evaluate** a limit **means** to **calculate the number that the limit is**.

On Page 40 of your text we see seven Properties of Limits listed. These seven properties can be thought of as the algebra of limits (where we think of "algebra" as a "set of rules for calculating").

The first two properties are important enough that we should look at them in some small detail.

1) $\lim_{x \to a}(k) =$ k constant, unchanging, unaffected by x!

2) $\lim_{x \to a}(x) =$ a

The five other properties allow us to "use" the above two.

> We will see that **using the Properties of Limits is a mathematical practice** very much like what you've been doing with algebra over the last few years. The **context** is just a little different. Instead of solving an equation, we are calculating potential functional values.

Section 1.5—Properties of Limits

The statement $\lim_{x\to a} f(x) = L$ says that the values of $f(x)$ become closer and closer to the number L as x gets closer and closer to the number a (from either side of a), such that $x \neq a$. This means that when finding the limit of $f(x)$ as x approaches a, there is no need to consider $x = a$. In fact, $f(a)$ need not even be defined. The only thing that matters is the behaviour of $f(x)$ near $x = a$.

EXAMPLE 1

Reasoning about the limit of a polynomial function

Find $\lim_{x\to 2} (3x^2 + 4x - 1)$.

Solution

It seems clear that when x is close to 2, $3x^2$ is close to 12, and $4x$ is close to 8. Therefore, it appears that $\lim_{x\to 2} (3x^2 + 4x - 1) = 12 + 8 - 1 = 19$.

In Example 1, the limit was arrived at intuitively. It is possible to evaluate limits using the following properties of limits, which can be proved using the formal definition of limits. This is left for more advanced courses.

Properties of Limits

For any real number a, suppose that f and g both have limits that exist at $x = a$.

1. $\lim_{x\to a} k = k$, for any constant k
2. $\lim_{x\to a} x = a$
3. $\lim_{x\to a} [f(x) \pm g(x)] = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
4. $\lim_{x\to a} [cf(x)] = c[\lim_{x\to a} f(x)]$, for any constant c
5. $\lim_{x\to a} [f(x)g(x)] = [\lim_{x\to a} f(x)][\lim_{x\to a} g(x)]$
6. $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, provided that $\lim_{x\to a} g(x) \neq 0$
7. $\lim_{x\to a} [f(x)]^n = [\lim_{x\to a} f(x)]^n$, for any rational number n

Example 1.5.1

Using the Properties of Limits, determine $\lim_{x\to 2}\left(\frac{3x^2-5x}{x+3}\right)$.

$$= \frac{\lim_{x\to 2}\left(3x^2-5x\right)}{\lim_{x\to 2}\left(x+3\right)} \quad \text{by property 6}$$

$$= \frac{\lim_{x\to 2}\left(3x^2\right) - \lim_{x\to 2}\left(5x\right)}{\lim_{x\to 2}\left(x\right) + \lim_{x\to 2}\left(3\right)} \quad \text{by property 3}$$

$$= \frac{3\lim_{x\to 2}\left(x^2\right) - 5\lim_{x\to 2}\left(x\right)}{2+3} \quad \text{property 4}$$

$$= \frac{3\cdot\left(\lim_{x\to 2}\left(x\right)\right)^2 - 5\cdot(2)}{5} \quad \text{property 7}$$

$$= \frac{3(2)^2-10}{5}$$

$$= \frac{2}{5}.$$

To be frank, it seems a little silly and certainly is tedious to evaluate limits by stating the various properties as we use them. In fact, using the various Properties of Limits can be boiled down to a single statement (or single Limit Law, if you will allow):

"a"

Plug it in, and see what happens

One of three things will happen:

1) You calculate a definite (a finite) number. ⇒ Hooray - you're done

Example 1.5.2

Evaluate $\lim\limits_{x\to -3}\left(\dfrac{2x-5}{7x}\right)$

$= \dfrac{11}{21}$

Jaiming -5×10^3.

2) Your calculation arrives at $\dfrac{\text{definite non-zero \#}}{0}$

You are done

The limit dne

Example 1.5.3

Evaluate $\lim\limits_{x\to 2}\left(\dfrac{3x^2-5}{x-2}\right)$

$= \text{"}\dfrac{7}{0}\text{"}$

dne

needs to be determined

3) Your calculation arrives at an **indeterminate** form: ie do more work!

$\frac{0}{0}$, or $\frac{\infty}{\infty}$, or $0 \cdot \infty$

have infinite possibility

What "more work"? Try factoring, Try conjugating, Try some 'trick'..

Example 1.5.4

Determine the limits, if they exist:

a) $\lim_{x \to 1}\left(\frac{2x^2+x-3}{x-1}\right)$ "$\frac{0}{0}$"

$$= \lim_{x \to 1}\left(\frac{(2x+3)\cancel{(x-1)}}{\cancel{x-1}}\right)$$

$$= 5$$

b) $\lim_{x \to 3}\left(\frac{\sqrt{x^2-5}-2}{x-3}\right)$ "$\frac{0}{0}$"

$$= \lim_{x \to 3}\left(\frac{\sqrt{x^2-5}-2}{x-3} \cdot \frac{\sqrt{x^2-5}+2}{\sqrt{x^2-5}+2}\right)$$

$$= \lim_{x \to 3}\left(\frac{(x^2-5)-4}{(x-3)\left(\sqrt{x^2-5}+2\right)}\right)$$

$$= \lim_{x \to 3}\left(\frac{x^2-9}{(x-3)\left(\sqrt{x^2-5}+2\right)}\right)$$ "$\frac{0}{0}$"

$$= \lim_{x \to 3}\left(\frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}\left(\sqrt{x^2-5}+2\right)}\right)$$

$$= \frac{6}{4} = \frac{3}{2}$$

cannot be dealt with

c) $\lim_{x\to 0}\left(\frac{(x+27)^{\frac{1}{3}}-3}{x}\right)$ " $\frac{0}{0}$ " More Work.

① Factoring a cube root is brutal

② $\sqrt[3]{x+27}$ ⇒ radical! maybe we can conjugate NO!

Trick - change the variable "x to u" EVERYWHERE

Let $u=(x+27)^{\frac{1}{3}}-3$ - Note as $x\to 0$ $u\to 0$

Need x by itself

$u+3=(x+27)^{\frac{1}{3}}$

$(u+3)^3=x+27$

$\Rightarrow x=(u+3)^3-27$

$("0"+27)^{\frac{1}{3}}-3$

$=27^{\frac{1}{3}}-3$

$=3-3$

$=0$

So we have

$$\lim_{u\to 0}\left(\frac{u}{\underbrace{(u+3)^3-27}_{\text{difference of cubes, so factor}}}\right) \quad \text{" }\frac{0}{0}\text{ " More work}$$

pattern

$a^3 \mp b^3$

$=(a \mp b)(a^2 \pm ab + b^2)$

$$=\lim_{u\to 0}\left(\frac{u}{((u+3)-3)((u+3)^2+3(u+3)+3^2)}\right)=\lim_{u\to 0}\left(\frac{\cancel{u}\ 1}{\cancel{u}((u+3)^2+3(u+3)+9)}\right)$$

$=\frac{1}{27}$

Class/Homework for Section 1.5

*Pg. 45 – 47 #3, 4, 7, 8abc (for #3, **explain** your thinking)*