1.1 Radical Expressions: Rationalizing

1. From the given binomial expressions, group any conjugate pairs and multiply them together. For any expression without its conjugate partner, write down the expression with its conjugate and multiply them together.

   $1+\sqrt{3},\ \ \sqrt{5}-2,\ \ \sqrt{7}+4,\ \ \sqrt{5}+2,\ \ \sqrt{3}-1,\ \ -4+\sqrt{7},\ \ 1-\sqrt{3}$

2. Rationalize the numerator and write in simplest form:

   a. $\dfrac{\sqrt{2}-1}{5}$ b. $\dfrac{3+\sqrt{5}}{-4}$ c. $\dfrac{\sqrt{3}-\sqrt{7}}{2\sqrt{3}}$ d. $\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{3}+\sqrt{2}}$ e. $\dfrac{\sqrt{2(x+h)}-\sqrt{2x}}{h}$

3. Rationalize the denominator and write in simplest form:

   a. $\dfrac{\sqrt{6}}{2\sqrt{15}}$ b. $\dfrac{-5}{1+\sqrt{3}}$ c. $\dfrac{1-\sqrt{2}}{1+\sqrt{2}}$ d. $\dfrac{2\sqrt{3}+3\sqrt{5}}{\sqrt{3}+\sqrt{5}}$ e. $\dfrac{2\sqrt{6}}{\sqrt{27}-2\sqrt{8}}$

Answers to selected problems:

2. a. $\dfrac{1}{5\left(\sqrt{2}+1\right)}$ c. $\dfrac{-2}{3+\sqrt{21}}$ e. $\dfrac{2}{\sqrt{2(x+h)}+\sqrt{2x}}$

3. a. $\dfrac{\sqrt{10}}{10}$ c. $\dfrac{3-2\sqrt{2}}{3}$ e. $-\dfrac{18\sqrt{2}+16\sqrt{3}}{5}$

1.2 The Slope of a Tangent

1. Simplify the Difference Quotients:

   a. $\dfrac{(8+h)^2-64}{h}$ b. $\dfrac{\frac{2}{3+h}-\frac{2}{3}}{h}$ c. $\dfrac{3(2+h)^3-24}{h}$ d. $\dfrac{2\sqrt{4+h}-4}{h}$ (hint: conjugate)

2. Determine, and simplify, an expression describing the slope of a secant through the given points:
   a. $P(2,3),\ Q\left(2+h,(2+h)^2-1\right)$
   b. $A(1, f(1)),\ B(1+h, f(1+h))$, where $f(x)=2x^3-1$
   c. $R(0,2),\ S\left(h,\sqrt{h+4}\right)$

3. Using a limit on the slope of a secant, determine the slope of the tangent to each curve at the given domain value (Don't forget – a domain value isn't enough info…you need a ***point***!):
   a. $f(x)=-2x^2+5$, at $x=1$
   b. $g(x)=-2x^2+5x$, at $x=1$
   c. $h(x)=\sqrt{2x+1}$, at $x=4$
   d. $p(x)=\dfrac{3}{x}$, at $x=2$

4. Determine the ***equation of the tangent*** to the curve $f(x)=2x^2+x+3$ at the point $(2,13)$. (Hint: a tangent is a line and so has an equation with a form like $y=mx+b$. Find m!)

5. A young Calculus student managed to lock himself in a room in a tower, 50*m* high. Looking out the window he notices a damsel of rescue and decides to get her attention by throwing a stone at her feet. The stone's height, as a function of time, is described by the function $h(t)=-4.9t^2-t+50$ (*h* is in *m*, and *t* is in seconds). Unfortunately for our student (who, let's face it, isn't very bright) the stone smashes through the windshield of a parked police car. Determine (DON'T BE AFRAID OF DECIMALS!!):
   a. The average velocity of the stone over the time interval $t\in[0,2]$.
   b. The instantaneous velocity of the stone at $t=2$ seconds.
   c. The velocity the stone hits the police car's windshield if the point of impact is 1*m* above ground (hint: you will need the time *t* when *h*(*t*) is 1*m*).

6. Determine the coordinates of the point on the curve $f(x) = -2x^2 + 3x$ where the tangent to $f(x)$ is parallel to the line $y = 5x + 2$ (*slope is your friend…do you see how friendly mathematics is*?!?).

7. Determine the equation of the line that passes through $(2,2)$ and is parallel to the tangent to the curve $f(x) = -3x^2 - 2x$ at $(-1,5)$.

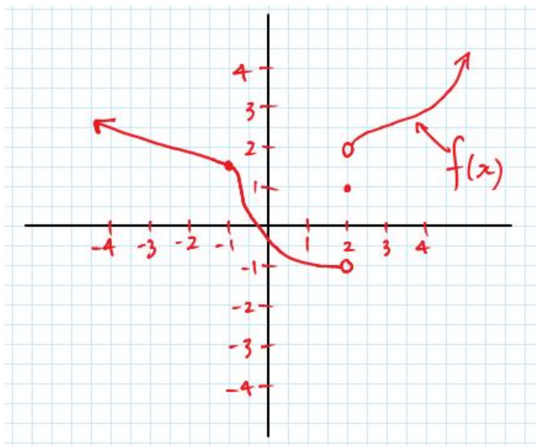
Answers to Selected Problems:

1. b. $-\dfrac{2}{3(3+h)}$ d. $\dfrac{2}{\sqrt{4+h}+2}$
2. b. $6 + 6h + 2h^2$
3. b. $m_{\tan} = 1$ d. $m_{\tan} = -\dfrac{3}{4}$
4. $y = 9x - 5$
5. a. $v_{avg} = -10.8m/s$ b. $v = -20.6m/s$ c. $t = 3.3s$, $v = -33.34m/s$
6. $(-0.5, -2)$
7. $y = 4x - 6$

1.4 The Limit of a Function

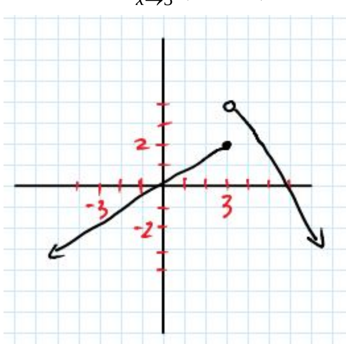
1. Given the sketch of the graph of $f(x)$, determine the following:

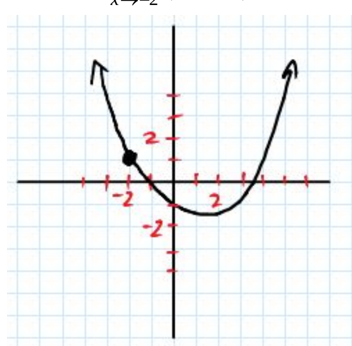
   a. $\lim_{x\to 1^-}\left(f(x)\right)$ b. $\lim_{x\to 2^+}\left(f(x)\right)$ c. $f(2)$ d. $\lim_{x\to 2^-}\left(f(x)\right)$

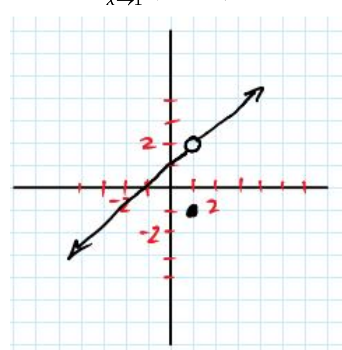


2. Using the given sketches, determine the limit (if it exists):

   a. $\lim_{x\to 3}\left(f(x)\right)$ b. $\lim_{x\to -2}\left(g(x)\right)$ c. $\lim_{x\to 1}\left(h(x)\right)$







3. Calculate the following limits:

   a. $\lim_{x\to -1}\left(x^2-2x-5\right)$ b. $\lim_{t\to 2}\left(\dfrac{3t-6}{5t+1}\right)$ c. $\lim_{x\to 4}\left(\sqrt{2x+1}\right)$

Answers to Selected Problems

1. d) -1 2. ASK for the answers 3. b) 0

1.4b Limit of a Function (continued)

1. Evaluate each limit:

   a. $\lim_{x\to3}(2x-1)$ b. $\lim_{t\to-2}(3t^2-5t)$ c. $\lim_{x\to2}(-3)$ d. $\lim_{x\to-2}(3^x)$ e. $\lim_{x\to1}\sqrt{\frac{2x+7}{5x}}$

2. Evaluate each limit. If the limit does not exist, explain why *(and it's not because the girls were mean)*.

   a. $\lim_{x\to-1^+}(3x-4)$ b. $\lim_{x\to2^+}\left(\frac{2x}{x-3}\right)$ c. $\lim_{x\to3}\left(\frac{2x}{x-3}\right)$ d. $\lim_{x\to0^-}\left(\frac{2x}{x-3}\right)$

3. Sketch the graph of each piecewise defined function and determine the indicated limit. If the limit does not exist, explain the problem:

   a. $f(x)=\begin{cases}x+1, & x<0\\ \cos(x), & x\geq0\end{cases}$; $\lim_{x\to0}(f(x))$

   b. $g(x)=\begin{cases}(x-1)^2+2, & x\leq1\\ 2x+1, & x>1\end{cases}$; $\lim_{x\to1}(g(x))$

   c. $p(t)=\begin{cases}6t, & t<0.5\\ 5, & t=0.5\\ \frac{3}{2t}, & t>0.5\end{cases}$; $\lim_{t\to0.5}(p(t))$

4. Sketch the possible graph of a function with the given characteristics:

   $\lim_{x\to-1^-}(f(x))=2,\ \lim_{x\to-1^+}(f(x))=1,\ f(-1)=2$

The following two problems are taken from the textbook Calculus and Vectors: Nelson, Pg. 39 #14, 15

5. Determine the real values of a, b, and c for the quadratic function $f(x)=ax^2+bx+c,\ a\neq0$, that satisfy the conditions:
   $f(0)=0,\ \lim_{x\to1}(f(x))=5$, and $\lim_{x\to-2}(f(x))=8$.

6. The fish population $P(t)$, in thousands, in a lake at time t, in years, is modelled by the population function:

$$P(t) = \begin{cases} 3 + \frac{1}{12}t^2, & 0 \le t \le 6 \\ 2 + \frac{1}{18}t^2, & 6 < t \le 12 \end{cases}$$

This function describes a sudden change in the population at time $t = 6$ years, due to a chemical spill.

a. Sketch the graph of $P(t)$.

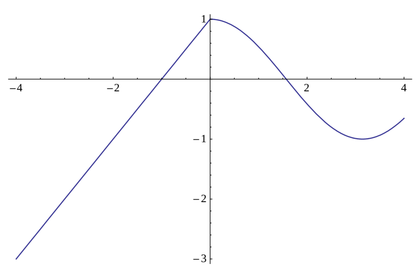
b. Evaluate $\lim_{t \to 6^-} (P(t))$ and $\lim_{t \to 6^+} (P(t))$.

c. How many fish died in the chemical spill?

d. How long after the spill did it take for the fish population to recover to the level before the spill?

Answers to Selected Problems:

1b) 22 d) $\frac{1}{9}$ 2a) -1 d) 0 3a)



$\lim_{x \to 0} (f(x)) = 1$

5) $a = 3, b = 2, c = 0$

1.5 Evaluating Limits

Don't forget: If you "***plug it in and see what happens***", and get "$\frac{0}{0}$", then **MORE WORK** needs to be done.

Note: you may find the following helpful: $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

Evaluate the following Limits:

a) $\lim_{x\to -2}\left(3x^2 - 5x + 1\right)$

b) $\lim_{x\to 2}\left(\sqrt{2x-1}\right)$

c) $\lim_{t\to 1}\left(\frac{3t}{t-1}\right)$

d) $\lim_{x\to 2}\left(\sqrt{x+\sqrt{x+7}}\right)$

e) $\lim_{x\to -2}\left(\sqrt{\frac{4x-1}{x+1}}\right)$

f) $\lim_{n\to -2}\left(\frac{3}{2-n}\right)^n$

g) $\lim_{x\to 3}\left(\frac{x^2-9}{x-3}\right)$

h) $\lim_{t\to 3}\left(\frac{\sqrt{x+1}-2}{x-3}\right)$

i) $\lim_{x\to -2}\left(\frac{x^3+8}{x+2}\right)$

j) $\lim_{m\to -2}\left(\frac{2m^2+3m-2}{3m^2-12}\right)$

k) $\lim_{x\to -3}\left(\frac{3-\sqrt{12+x}}{x+3}\right)$

l) $\lim_{x\to 2}\left(\frac{\sqrt{x+3}-\sqrt{7-x}}{x-2}\right)$

For the last three, use a change of variable to evaluate:

m) $\lim_{x\to 8}\left(\frac{x^{\frac{1}{3}}-2}{x-8}\right)$

n) $\lim_{x\to 1}\left(\frac{1-x}{x^{\frac{1}{3}}-1}\right)$

o) $\lim_{x\to 0}\left(\frac{(x+1)^{\frac{1}{3}}-1}{x}\right)$

Answers to selected problems:

b) $\sqrt{3}$ d) $\sqrt{5}$ f) $\frac{16}{9}$ h) $\frac{1}{4}$ i) 12 k) $-\frac{1}{6}$ m) $\frac{1}{12}$ o) $\frac{1}{3}$

1.6 Continuity

1. Determine all of the values of x for which the function is continuous. Be sure to explain your reasoning:
   a. $f(x)=4x^5-7x^3-5$
   b. $g(x)=\dfrac{2x^2+x-5}{2x-6x^2}$
   c. $h(x)=\sqrt{5-x}$
   d. $f(x)=\dfrac{3x^2-27}{x^2+1}$

2. Determine all the values of x where the function is continuous. Explain (using limits will be necessary to your explanation!):

$$f(x)=\begin{cases} 3x^2-5x+1, & x<1 \\ -\dfrac{2}{x+1}, & x\geq 1 \end{cases}$$

3. The given function is known to be continuous. Determine the value of k:

$$g(x)=\begin{cases} 2x-5, & x\leq 4 \\ \sqrt{x+2k}, & x>4 \end{cases}$$

4. Using interval notation state where the *signum* function is continuous. Sketch the function:

$$s(x)=\begin{cases} -1, & x<0 \\ 0, & x=0 \\ 1, & x>0 \end{cases}$$ (Note that *signum* has the word *sign* as its root)