1.4 The Limit of a Function (*Skipping 1.3*)

(*Geometric Point of View*)

Recall the definition of a function:

e.g. Given $f(x) = 3x^2 + 2$, then $f(2) =$

Consider the function $f(x) = \dfrac{x^2 - 9}{x - 3}$.

$f(3) =$???????

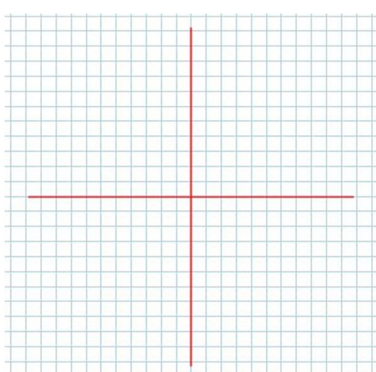
Now, we **can** calculate functional values such as:

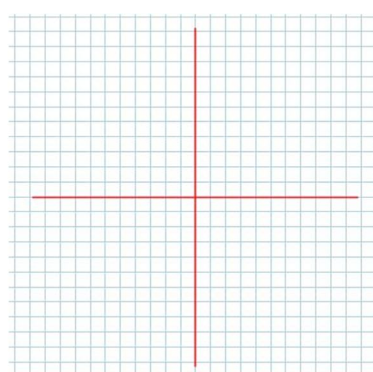
$f(2.99999999999999999997)$ or $f(3.0000000000000000000001)$, and these two functional values give hints to the **functional behaviour** of $f(x)$ **near** its problem domain value $x = 3$

Two possible functional behaviours of $f(x)$ at $x = 3$:

1)

2)





Definition 1.4.1

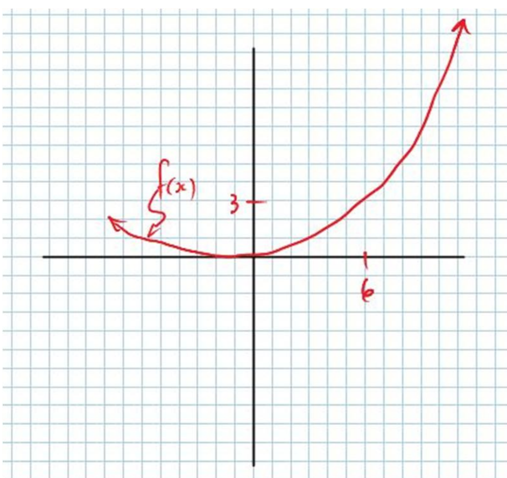
Given $y = f(x)$ we write

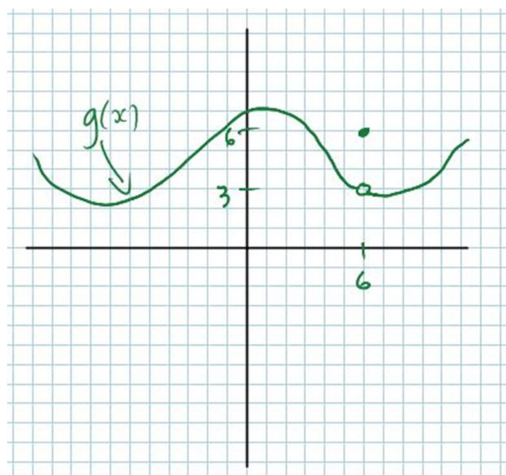
$$\lim_{x \to a} \left(f(x) \right) = L$$

to mean

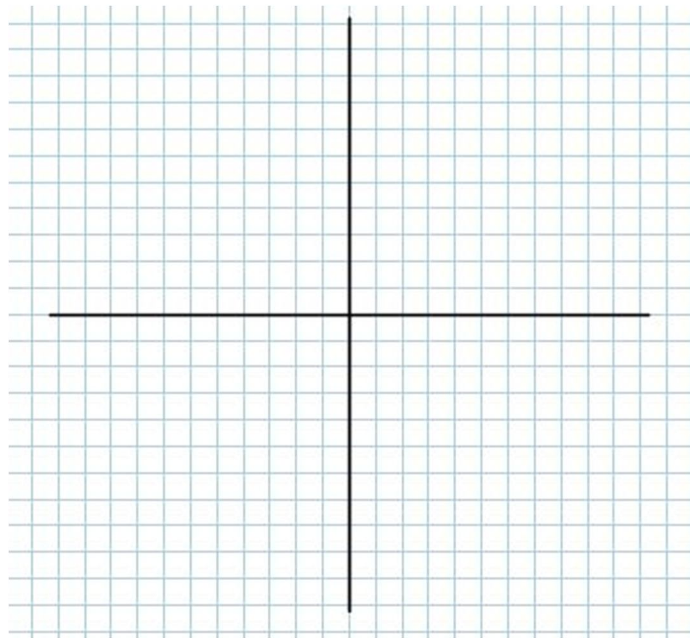
CAUTION:

Pictures





Example 1.4.1



Consider the sketch of the Piece-wise define function

$$f(x)=\begin{cases} x^2+1, & x\le 0 \\ x+2, & x>0 \end{cases}$$

Determine: a) $\lim_{x\to 2}\left(f(x)\right)$

b) $\lim_{x\to -1}\left(f(x)\right)$

c) $\lim_{x\to 0}\left(f(x)\right)$

We must consider **ONE SIDED LIMITS**

$\lim_{x \to 0^-} \left(f(x) \right)$

$\lim_{x \to 0^+} \left(f(x) \right)$

$$\therefore \lim_{x \to 0} \left(f(x) \right) =$$

Definition 1.4.2

Given a function $f(x)$, then

$$\lim_{x \to a} \left(f(x) \right) = L \text{ exists}$$

Thus, in **Example 1.4.1** c)

Note: We really only need to calculate one sided limits if:

1) We are finding a limit at a "**break-point**" of a piece-wise defined function.
2) At "**restrictions**" in domain values.

   e.g. for $f(x)=\sqrt{x}$,

   $\lim_{x \to 0^-}\left(f(x)\right)$ has no meaning, and so we can only consider $\lim_{x \to 0^+}\left(f(x)\right)$

Example 1.4.2

Calculate: a) $\lim_{x \to 3}(3x)$ b) $\lim_{x \to -2}\left(\frac{x^2}{4}\right)$

c) $\lim_{x \to \frac{5}{2}}\left(\frac{1}{2x-5}\right)$

To be continued…

Class/Homework for Section 1.4

Pg. 38 #6 – 8