

1.4b Limit of a Function (continued)

1. Evaluate each limit:

$$\text{a. } \lim_{x \rightarrow 3} (2x - 1) \quad \text{b. } \lim_{t \rightarrow -2} (3t^2 - 5t) \quad \text{c. } \lim_{x \rightarrow 2} (-3) \quad \text{d. } \lim_{x \rightarrow -2} (3^x) \quad \text{e. } \lim_{x \rightarrow 1} \sqrt{\frac{2x+7}{5x}}$$

2. Evaluate each limit. If the limit does not exist, explain why (*and it's not because the girls were mean*).

$$\text{a. } \lim_{x \rightarrow -1^+} (3x - 4) \quad \text{b. } \lim_{x \rightarrow 2^+} \left(\frac{2x}{x-3} \right) \quad \text{c. } \lim_{x \rightarrow 3} \left(\frac{2x}{x-3} \right) \quad \text{d. } \lim_{x \rightarrow 0^-} \left(\frac{2x}{x-3} \right)$$

3. Sketch the graph of each piecewise defined function and determine the indicated limit. If the limit does not exist, explain the problem:

$$\text{a. } f(x) = \begin{cases} x+1, & x < 0 \\ \cos(x), & x \geq 0 \end{cases}; \quad \lim_{x \rightarrow 0} (f(x))$$

$$\text{b. } g(x) = \begin{cases} (x-1)^2 + 2, & x \leq 1 \\ 2x+1, & x > 1 \end{cases}; \quad \lim_{x \rightarrow 1} (g(x))$$

$$\text{c. } p(t) = \begin{cases} 6t, & t < 0.5 \\ 5, & t = 0.5 \\ \frac{3}{2t}, & t > 0.5 \end{cases}; \quad \lim_{t \rightarrow 0.5} (p(t))$$

4. Sketch the possible graph of a function with the given characteristics:

$$\lim_{x \rightarrow -1^-} (f(x)) = 2, \quad \lim_{x \rightarrow -1^+} (f(x)) = 1, \quad f(-1) = 2$$

The following two problems are taken from the textbook *Calculus and Vectors: Nelson, Pg. 39 #14, 15*

5. Determine the real values of a , b , and c for the quadratic function

$f(x) = ax^2 + bx + c$, $a \neq 0$, that satisfy the conditions:

$$f(0) = 0, \quad \lim_{x \rightarrow 1} (f(x)) = 5, \quad \text{and} \quad \lim_{x \rightarrow -2} (f(x)) = 8.$$

6. The fish population $P(t)$, in thousands, in a lake at time t , in years, is modelled by the population function:

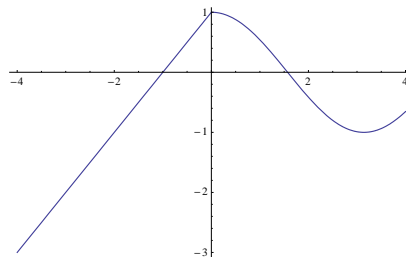
$$P(t) = \begin{cases} 3 + \frac{1}{12}t^2, & 0 \leq t \leq 6 \\ 2 + \frac{1}{18}t^2, & 6 < t \leq 12 \end{cases}$$

This function describes a sudden change in the population at time $t = 6$ years, due to a chemical spill.

- Sketch the graph of $P(t)$.
- Evaluate $\lim_{t \rightarrow 6^-} (P(t))$ and $\lim_{t \rightarrow 6^+} (P(t))$.
- How many fish died in the chemical spill?
- How long after the spill did it take for the fish population to recover to the level before the spill?

Answers to Selected Problems:

- 1b) 22 d) $\frac{1}{9}$ 2a) -1 d) 0 3a)
5) $a = 3, b = 2, c = 0$



$$\lim_{x \rightarrow 0} (f(x)) = 1$$