MCV4U 1.2 CHECK.

2. Determine, and simplify, an expression describing the slope of a secant through the given points:

   b. $A(1, f(1))$, $B(1+h, f(1+h))$, where $f(x) = 2x^3 - 1$

$$m_{sec} = \frac{f(1+h) - f(1)}{h}$$

$$= \frac{(2(1+h)^3 - 1) - (2(1)^3 - 1)}{h}$$

(plug in what you need)

$$= \frac{(2(1 + 3h + 3h^2 + h^3) - 1) - 1}{h}$$

$$= \frac{6h + 6h^2 + 2h^3}{h}$$

$$= \frac{h(6 + 6h + 2h^2)}{h}, \quad h \neq 0$$

$$= 6 + 6h + 2h^2$$

3. Using a limit on the slope of a secant, determine the slope of the tangent to each curve at the given domain value (Don't forget – a domain value isn't enough info…you need a ***point***!):

d. $p(x)=\dfrac{3}{x}$, at $x=2$

$p(2)=\dfrac{3}{2}$ (point $(2, 3/2)$)

$$m_{tan}=\lim_{h\to 0}\left(\frac{p(2+h)-p(2)}{h}\right)$$

$$=\lim_{h\to 0}\left(\frac{\frac{3}{2+h}-\frac{3}{2}}{h}\right)$$

$$=\lim_{h\to 0}\left(\frac{\frac{6-3(2+h)}{(2)(2+h)}}{h}\right)$$

$$=\lim_{h\to 0}\left(\frac{-3\cancel{h}}{2(2+h)}\cdot\frac{1}{\cancel{h}}\right)$$

$$=-\frac{3}{4}$$

5. A young Calculus student managed to lock himself in a room in a tower, 50*m* high. Looking out the window he notices a damsel of rescue and decides to get her attention by throwing a stone at her feet. The stone's height, as a function of time, is described by the function $h(t) = -4.9t^2 - t + 50$ (*h* is in *m*, and *t* is in seconds). Unfortunately for our student (who, let's face it, isn't very bright) the stone smashes through the windshield of a parked police car. Determine (DON'T BE AFRAID OF DECIMALS!!):
   a. The average velocity of the stone over the time interval $t \in [0,2]$.
   b. The instantaneous velocity of the stone at $t = 2$ seconds.
   c. The velocity the stone hits the police car's windshield if the point of impact is 1*m* above ground (hint: you will need the time *t* when *h*(*t*) is 1*m*).

a) $$V_{avg} = \frac{h(2) - h(0)}{2 - 0} = \frac{28.4 - 50}{2} = -10.8 \text{ m/sec}$$

b) $$V_{int} = \lim_{h \to 0} \left(\frac{h(2+h) - h(2)}{h} \right)$$

$$= \lim_{h \to 0} \left(\frac{(-4.9(2+h)^2 - (2+h) + 50) - 28.4}{h} \right)$$

$$= \lim_{h \to 0} \left(\frac{-19.8h - 4.9h^2 - h}{h} \right)$$

$$= \lim_{h \to 0} \left(\frac{\cancel{h}(-20.8 - 4.9h)}{\cancel{h}} \right) = -20.8 \text{ m/sec}$$

c) 1st: Solve eqn $-4.9t^2 - t + 50 = 1$

$$\Rightarrow -4.9t^2 - t + 49 = 0$$

$$\Rightarrow t = 3.06 \text{ sec}$$

$$2^{ND} \quad V_{inst} = \lim_{h\to 0}\left(\frac{h(3.06+h) - h(3.06)}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{-4.9(3.06+h)^2 - (3.06+h) + 50 \quad - \text{"}1\text{"}}{h}\right)$$

$$= -31 \text{ m/sec.}$$

6. Determine the coordinates of the point on the curve $f(x) = -2x^2 + 3x$ where the $\boxed{\text{tangent}}$ to $f(x)$ is parallel to the line $y = 5x + 2$ *(slope is your friend…do you see how friendly mathematics is?!?).*

↳ same slope

SLOPE.

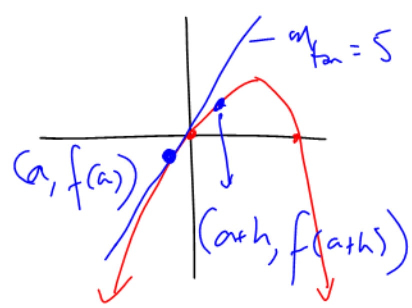
$y = 5x + 2$ has slope $\underline{\underline{5}}$

$\Rightarrow$ our tangent has slope 5

Slope of a TANGENT

$$m_{tan} = \lim_{h\to 0}(m_{sec})$$

$$m_{tan} = \lim_{h\to 0}\left(\frac{f(a+h) - f(a)}{h}\right)$$



$$\Rightarrow \quad 5 = \lim_{h\to 0}\left(\frac{\left(-2(a+h)^2 + 3(a+h)\right) - \left(-2a^2 + 3a\right)}{h}\right)$$

$\Downarrow$

$$5 = \lim_{h \to 0} \left(\frac{-2(a^2 + 2ah + h^2) + 3a + 3h + 2a^2 - 3a}{h} \right)$$

$$\Rightarrow 5 = \lim_{h \to 0} \left(\frac{\cancel{h}(-4a - 2h + 3)}{\cancel{h}} \right)$$

$$\Rightarrow \quad 5 = -4a + 3$$

$$\Rightarrow \quad a = -0.5$$

$\therefore$ The point $(a, f(a)) = (-0.5, f(-0.5))$

$= (-0.5, -2)$

7. Determine the equation of the line that passes through $(2,2)$ and is parallel to the tangent to the curve $f(x) = -3x^2 - 2x$ at $(-1,5)$. Whoops! $(-1,5)$ is NOT on $f(x)$

$(-1,5)$ is on $f(x) = +3x^2 - 2x$

The line we are looking for has the same slope as the tangent to $f(x)$ at $x = -1$

$$m_{tan} = \lim_{h \to 0} \left(\frac{f(-1+h) - f(-1)}{h} \right) = \text{work} = 4$$

$\therefore$ our line has eqn (slope-point) $y - y_1 = m(x - x_1)$, where $(x_1, y_1) = (2,2)$

$y - 2 = 4(x - 2) \Rightarrow y = 4x - 6$