1.4 The Limit of a Function (*Skipping 1.3*)

(*Geometric Point of View*)

Recall the definition of a function: $\Rightarrow$ number generator giving POINTS $(x, f(x))$

An algebraic rule which assigns exactly one value in the range to each value in the domain.

e.g. Given $f(x) = 3x^2 + 2$, then $f(2) =$

$$f(2) = 14$$

Consider the function $f(x) = \dfrac{x^2 - 9}{x - 3}$.

There is no $f(x)$ value for $x = 3$

$f(3) =$???????

$x = 3 \notin D_f$

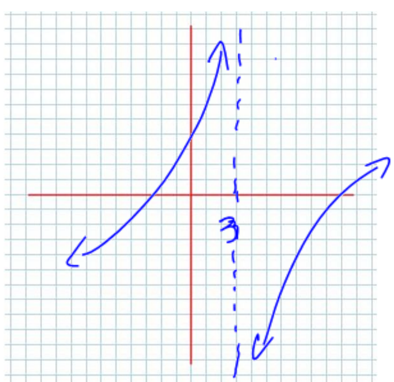
Now, we **can** calculate functional values such as:

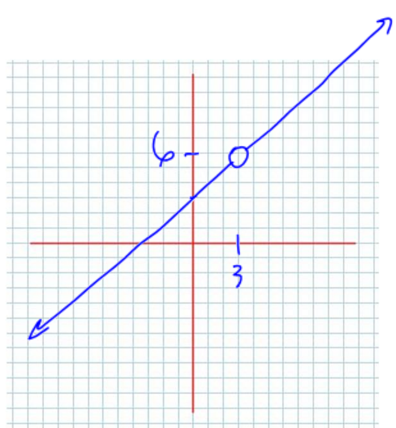
$f(2.9999999999999999997)$ or $f(3.0000000000000000000001)$, and these two functional values give hints to the **functional behaviour** of $f(x)$ **near** its "problem domain value" $x = 3$

Two possible functional behaviours of $f(x)$ at $x = 3$:

1) Vertical Asymptote

2) Hole.





Definition 1.4.1

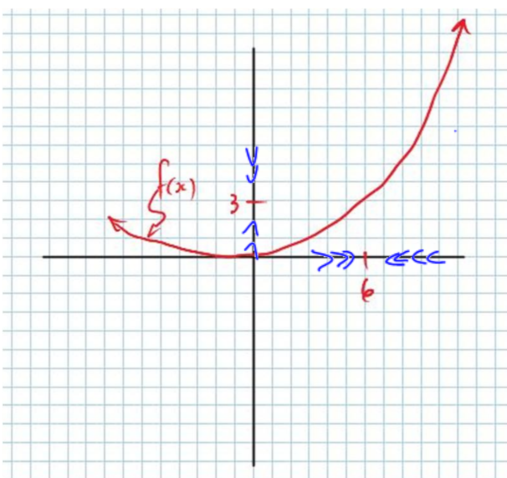
Given $y = f(x)$ we write

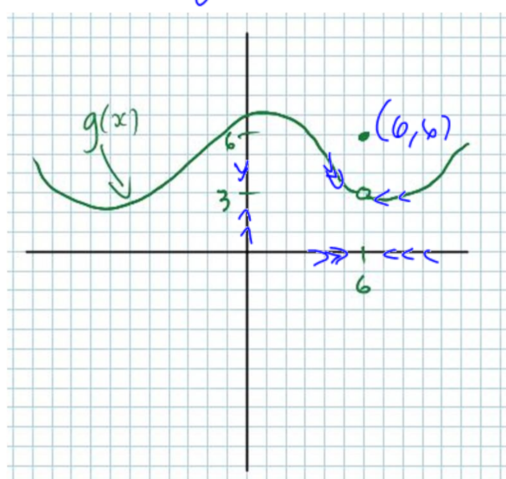
$$\lim_{x \to a} \left(f(x) \right) = L$$

to mean as x gets RIDICULOUSLY close to the value (number) "a", $f(x)$ is FORCED to get RIDICULOUSLY close to the value L. We call L a limit

CAUTION: x can approach "a" from two sides!!

Pictures What is happening to $f(x)$ & $g(x)$ as $x \to 6$





$g(6) = 6$

$$\lim_{x \to 6} \left(f(x) \right) = 3 \qquad \lim_{x \to 6} \left(g(x) \right) = 3$$

Example 1.4.1 $f(0) = 1$

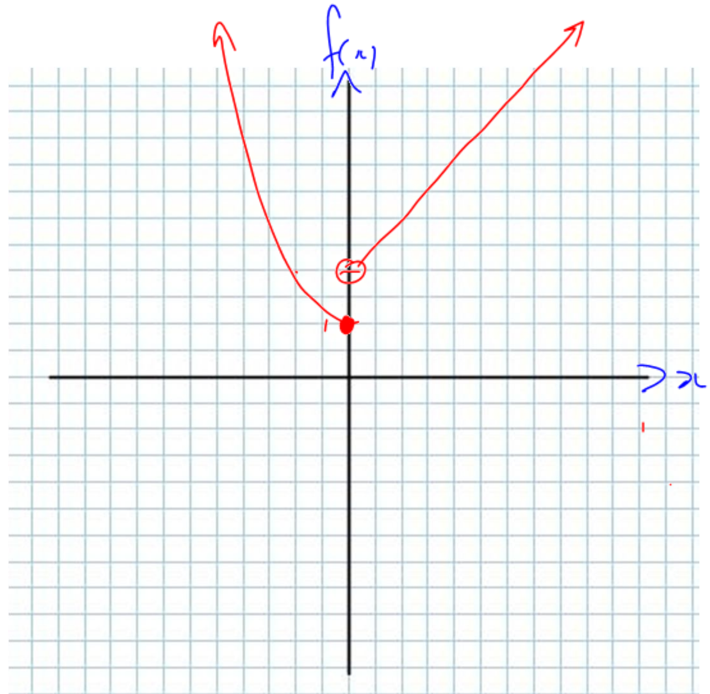
Consider the sketch of the Piece-wise define function

$$f(x) = \begin{cases} x^2 + 1, & x \le 0 \\ x + 2, & x > 0 \end{cases}$$

Determine: a) $\lim_{x \to 2}(f(x))$

b) $\lim_{x \to -1}(f(x))$

c) $\lim_{x \to 0}(f(x))$



a) $\lim_{x \to 2}(f(x))$

$= \lim_{x \to 2}(x + 2)$

$= 4$

b) $\lim_{x \to -1}(f(x))$

$= \lim_{x \to -1}(x^2 + 1)$

$= 2$

c) $\lim_{x \to 0}(f(x))$

PROBLEM! WHICH fn behavior behavior "x^2+1" or "$x+2$" do we use to represent $f(x)$? we can approach $x=0$ from two sides!!!!!

We must consider **ONE SIDED LIMITS**

Notation: $\lim_{x \to a^-} (f(x))$ means $x \to a$ "from below"

$\lim_{x \to a^+} (f(x))$ " " "from above"

$\lim_{x \to 0^-} (f(x))$ $\qquad\qquad$ $\lim_{x \to 0^+} (f(x))$

$\lim_{x \to 0^-} (x^2 + 1)$ $\qquad\qquad$ $\lim_{x \to 0^+} (x + 2)$

$= 1$ $\qquad\qquad$ $= 2$

$\therefore \lim_{x \to 0} (f(x)) =$ does not exist!

Definition 1.4.2

Given a function $f(x)$, then

$\lim_{x \to a} (f(x)) = L$ exists iff $\Rightarrow$ if and only if ($\Leftrightarrow$)

$\lim_{x \to a^-} (f(x)) = L$ and $\lim_{x \to a^+} (f(x)) = L$, where

L is a finite number.

Thus, in **Example 1.4.1** c)

$\lim_{x \to 0} (f(x))$ d. n. e. $\Leftarrow$ does not exist

Note: We really only need to calculate one sided limits if:

1) We are finding a limit at a "**break-point**" of a piece-wise defined function.
2) At "**restrictions**" in domain values.

   e.g. for $f(x)=\sqrt{x}$,

   $\lim\limits_{x\to 0^-}\left(f(x)\right)$ has no meaning, and so we can only consider $\lim\limits_{x\to 0^+}\left(f(x)\right)$

Example 1.4.2

Calculate: a) $\lim\limits_{x\to 3}(3x)$ b) $\lim\limits_{x\to -2}\left(\dfrac{x^2}{4}\right)$

$= 9$

$= +1$

c) $\lim\limits_{x\to \frac{5}{2}}\left(\dfrac{1}{2x-5}\right)$

$x = 5/2$ is a restriction

$\Rightarrow$ 2 one sided limits

$$\lim_{x\to \frac{5}{2}^-}\left(\frac{1}{2x-5}\right) = \text{“}\frac{1}{-0}\text{”} = -\infty$$

$$\lim_{x\to \frac{5}{2}^+}\left(\frac{1}{2x-5}\right) = \frac{1}{\text{“}+0\text{”}} = +\infty$$

$$\therefore \lim_{x\to 5/2}\left(\frac{1}{2x-5}\right)$$

d.n.e.

To be continued…

Class/Homework for Section 1.4

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