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MATH@TD

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MCV4U – W14

NAME ***Solutions***

Quist: Chapter 1 – Introduction to Calculus
(*Note: You will receive a Communications grade out of 5 for how well your math is presented*)

1. Determine the equation of the tangent to the function $f(x)=4\sqrt{x-3}$ at the point $(4,4)$.

K____/3

(*Note: We need to find the equation of a line* $y = mx + b$. *The slope is given by the tangent slope "formula", and we use the given point for b*)

$$\begin{aligned}
m_{\tan} &= \lim_{h\to 0}\left(\frac{f(4+h)-f(4)}{h}\right)\\
&= \lim_{h\to 0}\left(\frac{4\sqrt{(4+h)-3}-4}{h}\right)\\
&= \lim_{h\to 0}\left(\frac{4\sqrt{1+h}-4}{h}\cdot\frac{4\sqrt{1+h}+4}{4\sqrt{1+h}+4}\right)\\
&= \lim_{h\to 0}\left(\frac{16(1+h)-16}{h\left(4\sqrt{1+h}+4\right)}\right)\\
&= \lim_{h\to 0}\left(\frac{16h}{h\left(4\sqrt{1+h}+4\right)}\right)\\
&= \lim_{h\to 0}\left(\frac{16}{4\sqrt{1+h}+4}\right)\\
&= 2
\end{aligned}$$

So, we have for the equation of the tangent $y=2x+b$. To find b we can use the point $(4,4)$:

$$\begin{aligned}
&4=2(4)+b\\
&\Rightarrow b=-4\\
&\Rightarrow y=2x-4
\end{aligned}$$

2. Determine the limit for each problem, if the limit exists: **A___/1, ___/2, ___/3, ___/2**

a) $\lim_{x\to 3}\left(\frac{x^2-4}{x-2}\right)=5$

b) $\lim_{x\to 3}\left(\frac{2x^2-5x-3}{(x-3)^2}\right)$ $\frac{0}{0}$

$=\lim_{x\to 3}\left(\frac{(2x+1)(x-3)}{(x-3)^2}\right)$

$=\lim_{x\to 3}\left(\frac{2x+1}{x-3}\right)$

$d.n.e.$

c)

$\lim_{x\to 2}\left(\frac{x-2}{\sqrt{4x+1}-3}\right)$ $\frac{0}{0}$

$=\lim_{x\to 2}\left(\frac{x-2}{\sqrt{4x+1}-3}\cdot\frac{\sqrt{4x+1}+3}{\sqrt{4x+1}+3}\right)$

$=\lim_{x\to 2}\left(\frac{(x-2)(\sqrt{4x+1}+3)}{(4x+1)-9}\right)$

$=\lim_{x\to 2}\left(\frac{(x-2)(\sqrt{4x+1}+3)}{4(x-2)}\right)$

$=\frac{3}{2}$

d)

$\lim_{x\to -1}\left(\frac{x+1}{x^3+1}\right)$ $\frac{0}{0}$

$=\lim_{x\to -1}\left(\frac{x+1}{(x+1)(x^2-x+1)}\right)$

$=\lim_{x\to -1}\left(\frac{1}{x^2-x+1}\right)$

$=\frac{1}{3}$

3. a) State the three conditions which make the function $f(x)$ continuous at the single value $x = a$. **K____/1**

b) Determine all values of x for which the function **T____/4**

$$g(x)=\begin{cases}3x^2-2x, & x\le 1\\ \sqrt{2x-1}, & x>1\end{cases}$$ is continuous.

a) 1. $f(a)$ exists

2. $\lim_{x\to a}(f(x))$ exists

3. $\lim_{x\to a}(f(x)) = f(a)$

b) On $(-\infty,1)$ $g(x)$ is a polynomial and so continuous

On $(1,\infty)$ $g(x)$ is a radical function (continuous on $[1/2,\infty)$) and so cts

$\therefore g(x)$ is cts on $(-\infty,1)\cup(1,\infty)$ $(*)$

At $x=1$ we consider single value continuity: (3 conditions)

1. $g(1)=1$ (and so the functional value exists)

2. We must determine $\lim_{x\to 1}(g(x))$

(which requires two one sided limits because of the behaviour change at $x=1$)

$$\lim_{x\to 1^-}(g(x)) = \lim_{x\to 1^-}(3x^2-2x) = 1$$

$$\lim_{x\to 1^+}(g(x)) = \lim_{x\to 1^+}\left(\sqrt{2x-1}\right) = 1$$

$$\therefore \lim_{x\to 1}(g(x)) = 1$$

3. $\because \lim_{x\to 1}(g(x)) = g(1)$

$\therefore g(x)$ is cts at $x=1$

$\because g(x)$ is also cts on $(-\infty,1)\cup(1,\infty)$ (by $(*)$)

$\therefore g(x)$ is continuous on $(-\infty,\infty)$