2.2 Derivatives of Polynomial Functions

Using the "formal" definition of the derivative

$$\lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

can be painful and tedious. Mathematicians, always wanting to reduce pain and tedium develop rules to simply our work. And we will spend a bit of time learning the rules, and how to use them.

The Derivative of a Constant Function

Given $f(x)=k$, **then** $f'(x)=0$

Proof:

KEEP IN MIND
The derivative is a ***TOOL*** *for measuring rate of change. In terms of algebraic functions, the* ***derivative calculates the slope of tangents*****.**

The Derivative of a Constant times a Differentiable Function

Given a differentiable function, $f(x)$, **then**, the function $g(x) = k \cdot f(x)$, k constant, is also differentiable, and $g'(x) = k \cdot f'(x)$.

Proof:

The Derivative of a Power Function (*The Power Rule*)

Given a power function $f(x) = x^n$, **then** $f(x)$ is differentiable and $f'(x) = n \cdot x^{n-1}$
(*See pg. 77 for a proof, which requires knowledge of the Binomial Theorem*)

The Power Rule says:
"***Bring the exponent down, and reduce the exponent by 1***"

Example 2.2.1

Differentiate

a) $f(x) = x^3$ b) $g(x) = x^{-4}$ c) $h(x) = x^{\frac{2}{5}}$

d) $f(x) = 7x^4$ e) $g(x) = x$ f) $h(x) = -6x$

g) $f(x) = \pi x^5$ h) $g(x) = \dfrac{3}{x^6}$ i) $h(x) = \sqrt[3]{x}$

The Derivatives of Sums and Differences of Differentiable Functions

Given differentiable functions $f(x)$ and $g(x)$, then the functions

$$F(x) = f(x) + g(x), \text{ and}$$
$$G(x) = f(x) - g(x)$$

are also differentiable and

$$F'(x) = f'(x) + g'(x), \text{ and}$$
$$G'(x) = f'(x) - g'(x)$$

See page 79 for the simple proofs of these results.

Example 2.2.2

Differentiate $f(x) = 3x^3 - 4\sqrt{x} + \dfrac{7}{x^2}$

Example 2.2.3

Differentiate $g(x) = \dfrac{7x^2 - 5x^3 + 8x}{\sqrt{x}}$

Class/Homework for Section 2.2

Pg. 82 – 84 #2 – 4, 6, 7, 9, 11 – 14, 16, 18, 21, 23, 25