2.5 Derivatives of Composite Functions

Thus far we have seen a number of simplifying rules for determining derivatives of various functions. We now turn to differentiating composite functions using the simplifying rule known as **The Chain Rule**.

Recall that given two functions $f(x)$ and $g(x)$, then the composition of $f(x)$ with $g(x)$ is defined to be:

$$\begin{aligned} F(x) &= (f \circ g)(x) \\ &= f(g(x)) \end{aligned}$$

Example 2.5.1

Given $F(x) = \sqrt{3x^2 - 5x + 1}$ determine two functions $f(x)$ and $g(x)$ so that $f(g(x)) = F(x)$. Also determine the composite function $G(x) = (g \circ f)(x)$.

The Chain Rule

Given two differentiable functions $f(x)$ and $g(x)$, then the composite function

$$F(x) = f(g(x))$$

is also differentiable, and

$$\frac{dF(x)}{dx} = \frac{df(g(x))}{dx} \cdot \frac{dg(x)}{dx}, \text{ or } F'(x) = f'(g(x)) \cdot g'(x)$$

In "words":

Example 2.5.2

Differentiate:

a) $f(x) = \left(5x^2 - 7x + \sqrt{x}\right)^{97}$

b) $g(x) = \sqrt{3x^2 - 5x + 1}$

c) $h(x) = \left(\dfrac{x+2}{3x-1}\right)^{12}$

Example 2.5.3 (*A typical example using Leibniz Notation*)

Given that $V(x) = 3x^2 + 2x - 7,$ and that $x = 3\sqrt{t}$, determine:

a) $\dfrac{dV}{dt}$

b) $\dfrac{dV}{dt}$ when $t = 9$

Example 2.5.4

Determine the equation of the tangent to $y=(3x+2)^{\frac{1}{3}}$ at $x=2$

Class/Homework for Section 2.5

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