2.5 Derivatives of Composite Functions (The Chain Rule)

1. Differentiate the following:
    a. $f(x)=\left(2x^3-5x^2+3\sqrt{x}-1\right)^{13}$
    b. $h(x)=\dfrac{2}{\left(3x^2-5x\right)^4}$
    c. $g(x)=\sqrt{-2x^3+\pi^2}$
    d. $f(x)=\left(\dfrac{x^2-7}{x^3+1}\right)^4$
    e. $g(x)=\dfrac{3x^2}{\sqrt{3-4x}}$
    f. $f(x)=\sqrt{x+\sqrt{3x-1}}$
    g. $p(t)=\left(3t^2+5t\right)\sqrt{2t^{-3}+3t}$

2. For what values of *x* do the given functions have the same slope?
   $f(x)=(2x-3)^3,\quad g(x)=-3(x-2)^2$

3. Determine the equation of the tangent to the curve $f(x)=\sqrt[3]{x+64}$ at $x=0$.

4. Using the Chain Rule and Leibniz Notation find $\dfrac{dy}{dx}$ at the given value of *x*.
    a. $y=2t^3-5t,\ t=3\sqrt{x},\ x=2$ (If you choose to answer in a decimal form, round to two decimal places)
    b. $y=(3u+5)^3+2\left(5-u^2\right)^2,\ u=\dfrac{x+1}{2x-3},\ x=1$

5. A $50\,000$ L tank can be drained in 30 minutes. The volume of water remaining in the tank after *t* minutes of draining is modelled by $V(t)=50\,000\left(1-\dfrac{t}{30}\right)^2,\ 0\le t\le 30$. At what rate, rounded to the nearest two decimals, is water flowing at $t=10$ min?
   (*Taken from the Nelson Text: Pg. 106 #15*)

Answers to Selected Problems on the reverse

1b) $h'(x) = \frac{-8(6x-5)}{(3x^2-5x)^5}$ d) $f'(x) = \frac{-4x(x^2-7)^3(x^3-21x-2)}{(x^3+1)^5}$

2. $x = \frac{7}{4}$, or $x = 1$

4b) $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$, for $u = -2$ and $x = 1$

$= -125$

5. $-2,222.22 \ L/\text{min}$ (Where the negative means that water is flowing out of the tank.)