Hwk Check

3. A ball is moving through space and its **position** is described by the function $s(t) = -4.9t^2 + t + 5$. Determine the ball's **velocity** at times $t = 0,\ 2,\ 3$ seconds. (Hint: it's easiest if you find velocity as a function of time).

Note: velocity is $\frac{\text{change in position}}{\text{change in time}}$

$$s'(t) = \lim_{h \to 0} \left(\frac{s(t+h) - s(t)}{h} \right)$$

$$= \lim_{h \to 0} \left(\frac{(-4.9(t+h)^2 + (t+h) + 5) - (-4.9t^2 + t + 5)}{h} \right)$$

ie $s'(t) = \frac{ds}{dt} = v(t)$

$$= \lim_{h \to 0} \left(\frac{-9.8th - 4.9h^2 + h}{h} \right)$$

(skipped some steps)

$$\boxed{s'(t) = -9.8t + 1}$$

ie velocity is

$v(t) = -9.8t + 1$

velocity is the derivative position (with respect to time)

$\therefore$ at $t = 0$ $\quad v(0) = 1$ m/sec

at $t = 2$ $\quad v(2) = -9.8(2) + 1 = -18.6$ m/sec

at $t = 3$ $\quad v(3) = -9.8(3) + 1 = -28.4$ m/sec

4. Determine the equation of the tangent to the function $g(x)=\sqrt{x-1}$, and which is parallel to the line with equation $y=\frac{1}{4}x-1$.

tangent = derivative | we want our derivative = $\frac{1}{4}$

$$g'(x) = \lim_{h\to 0}\left(\frac{g(x+h)-g(x)}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{\sqrt{x+h-1}-\sqrt{x-1}}{h}\cdot\frac{\sqrt{x+h-1}+\sqrt{x-1}}{\sqrt{x+h-1}+\sqrt{x-1}}\right)$$

$$= \lim_{h\to 0}\left(\frac{(x+h-1)-(x-1)}{h\left(\sqrt{x+h-1}+\sqrt{x-1}\right)}\right)$$

$$= \lim_{h\to 0}\left(\frac{\cancel{h}\ 1}{\cancel{h}\left(\sqrt{x+h-1}+\sqrt{x-1}\right)}\right)$$

$$= \frac{1}{2\sqrt{x-1}}$$

(the derivative is a slope finding fn! we want $g'(x)=\frac{1}{4}$)

$$\Rightarrow \frac{1}{4} = \frac{1}{2\sqrt{x-1}}$$ (solve for x)

$$\Rightarrow 2\sqrt{x-1} = 4$$

$$\sqrt{x-1} = 2$$

$$\Rightarrow x-1 = 4$$

$$x = 5$$

(so at the point $(5, g(5)) = (5, 2)$ the tangent has a slope of $\frac{1}{4}$)

$\therefore$ The eqn of the tangent is

$$y = mx + b \quad , \quad m = \frac{1}{4} \text{ at the point } (5,2)$$

$$\Rightarrow y = \frac{1}{4}x + b$$

$$\Rightarrow 2 = \frac{1}{4}(5) + b$$

$$\Rightarrow b = \frac{3}{4} \qquad \left(\text{from } \frac{8}{4} = \frac{5}{4} + b \right)$$

$$\therefore y = \frac{1}{4}x + \frac{3}{4} \text{ is eqn}$$

$$\left(x - 4y + 3 = 0 \text{ is standard form} \right)$$

2.2 Derivatives of Polynomial Functions

Using the "formal" definition of the derivative

$$f'(x) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

can be painful and tedious. Mathematicians, always wanting to reduce pain and tedium develop rules to simply our work. And we will spend a bit of time learning the rules, and how to use them.

The Derivative of a Constant Function

Given $f(x)=k$, **then** $f'(x)=0$

Proof:

$$f'(x) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{k-k}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{0}{h}\right)$$

$$= \lim_{h\to 0}(0)$$

$$= 0 \quad \square$$

KEEP IN MIND
The derivative is a **TOOL** *for measuring rate of change. In terms of algebraic functions, the* ***derivative calculates the slope of tangents***.

(ie derivatives measure "rate of change!")

means $f'(x)$ exists!

and $f'(x) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$

The Derivative of a Constant times a Differentiable Function

Given a differentiable function, $f(x)$, **then**, the function $g(x) = k \cdot f(x)$, k constant, is also differentiable, and $g'(x) = k \cdot f'(x)$.

we need to show 2 things

① show $g'(x)$ exists

ie show $\lim_{h\to 0}\left(\frac{g(x+h)-g(x)}{h}\right)$ exists

② show $g'(x) = k \cdot f'(x)$

Proof:

$$g'(x) = \lim_{h\to 0}\left(\frac{g(x+h)-g(x)}{h}\right)$$

$$= \lim_{h\to 0}\left(\frac{k\cdot f(x+h) - k\cdot f(x)}{h}\right)$$

$$= k \cdot \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$ — exists! (by hypothesis)

$$= k\cdot f'(x)$$ $\Rightarrow$ this exists! $\therefore$ $g'(x)$ exists!

and

$$g'(x) = k\cdot f'(x)$$

□

The Derivative of a Power Function (*The Power Rule*)

Given a power function $f(x)=x^n$, **then** $f(x)$ is differentiable and $f'(x)=n\cdot x^{n-1}$
(*See pg. 77 for a proof, which requires knowledge of the Binomial Theorem*)

The Power Rule says:
"***Bring the exponent down, and reduce the exponent by 1***"

Example 2.2.1

Differentiate

a) $f(x)=x^3$

$f'(x)=3x^2$

b) $g(x)=x^{-4}$

$-4-1=-5$

$g'(x)=-4x^{-5}$

c) $h(x)=x^{\frac{2}{5}}$

$\frac{2}{5}-1=\frac{2}{5}-\frac{5}{5}$
$=-3/5$

$h'(x)=\frac{2}{5}x^{-3/5}$

d) $f(x)=7x^4$

differentiable fn

$f'(x)=7(x^4)'$
$=7(4x^3)$
$=28x^3$

e) $g(x)=x^1$

$g'(x)=1\cdot x^0$
$=1$

f) $h(x)=-6x$

$h'(x)=-6$

g) $f(x)=\pi x^5$

$f'(x)=5\pi x^4$

h) $g(x)=\frac{3}{x^6}$

not a power fn

write as a power fn

$g(x)=3x^{-6}$

$g'(x)=-18x^{-7}$

$\left(=-\frac{18}{x^7}\right)$

i) $h(x)=\sqrt[3]{x}$

$h(x)=x^{\frac{1}{3}}$

$\therefore h'(x)=\frac{1}{3}x^{\frac{1}{3}-1}$

$=\frac{1}{3}x^{-2/3}$

The Derivatives of Sums and Differences of Differentiable Functions

Given differentiable functions $f(x)$ and $g(x)$, then the functions

$$F(x) = f(x) + g(x), \text{ and}$$
$$G(x) = f(x) - g(x)$$

are also differentiable and

$$F'(x) = f'(x) + g'(x), \text{ and}$$
$$G'(x) = f'(x) - g'(x)$$

differentiate "term by term"

See page 79 for the simple proofs of these results.

Example 2.2.2

Differentiate $f(x) = 3x^3 - 4\sqrt{x} + \dfrac{7}{x^2}$

rewrite as power fns!

$\Rightarrow f(x) = 3x^3 - 4x^{\frac{1}{2}} + 7x^{-2}$

$\therefore f'(x) = 9x^2 - 2x^{-\frac{1}{2}} - 14x^{-3} = 9x^2 - \dfrac{2}{\sqrt{x}} - \dfrac{14}{x^3}$

Example 2.2.3

Differentiate $g(x) = \dfrac{7x^2 - 5x^3 + 8x}{\sqrt{x}}$

this is common to each term in the numerator!

rewrite $g(x)$ as a sum/difference of power fns

$\Rightarrow g(x) = \dfrac{7x^2}{x^{\frac{1}{2}}} - \dfrac{5x^3}{x^{\frac{1}{2}}} + \dfrac{8x}{x^{\frac{1}{2}}}$

$= 7x^{\frac{3}{2}} - 5x^{\frac{5}{2}} + 8x^{\frac{1}{2}}$

$2 - \frac{1}{2}$
$= \frac{4}{2} - \frac{1}{2}$
$= \frac{3}{2}$

$\therefore g'(x) = \dfrac{21}{2}x^{\frac{1}{2}} - \dfrac{25}{2}x^{\frac{3}{2}} + 4x^{-\frac{1}{2}}$

Class/Homework for Section 2.2

Pg. 82 – 84 #2 – 4, 6, 7, 9, 11 – 14, 16, 18, 21, 23, 25