

2.3b More Product Rule Examples

Recall that the product rule says that given two differentiable functions $f(x)$ and $g(x)$, then the “product function” $F(x) = f(x) \cdot g(x)$ is also differentiable, and

$$\frac{dF}{dx}(x) = \frac{df}{dx}(x) \cdot g(x) + f(x) \cdot \frac{dg}{dx}(x) \text{ or, } F'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

Example 2.3.5

Differentiate, and simplify $f(t) = (3t^2 - 2t)(5t^3 - 2t^2 + 1)^3$

Example 2.3.6

Determine the derivative of $h(x) = \frac{3x-2}{5x^2+1}$

$$h(x) = (3x-2)(5x^2+1)^{-1}$$

$$h'(x) = (3)(5x^2+1)^{-1} + (3x-2)(-1)(5x^2+1)^{-2}(10x)$$

$$h'(x) = \frac{3}{5x^2+1} + \frac{(3x-2)(-1)(10x)}{(5x^2+1)^2}$$

$$= \frac{1}{5x^2+1} \left(\frac{3(5x^2+1) - (3x-2)(10x)}{(5x^2+1)} \right)$$

$$= \frac{1}{5x^2+1} \left(\frac{(15x^2+3) - (30x^2-20x)}{5x^2+1} \right)$$

$$= \left(\frac{-15x^2 + 20x + 3}{(5x^2 + 1)^2} \right)$$

Example 2.3.7

Differentiate $s(t) = 3t^2(2t - 5)$

$$\begin{aligned} s'(t) &= 6t(2t - 5) + 3t^2(2) \\ &= 6t(2t - 5) + 6t^2 \\ &= 6t(2t - 5) + 6t^2 \\ &= 6t(3t - 5) \end{aligned}$$

Example 2.3.8

Determine the slope of a tangent to $f(x) = 2x^3(3x^2 - 5x + 1)^4$ at $x = 1$

Class/Homework for Section 2.3b

Pg. 90 – 91 #7 – 10, Pg. 92 #6 – 8