2.3b More Product Rule Examples

Recall that the product rule says that given two differentiable functions $f(x)$ and $g(x)$, then the "product function" $F(x) = f(x)\cdot g(x)$ is also differentiable, and

$$\frac{dF}{dx}(x) = \frac{df}{dx}(x)\cdot g(x) + f(x)\cdot\frac{dg}{dx}(x) \text{ or, } F'(x) = f'(x)\cdot g(x) + f(x)\cdot g'(x)$$

Example 2.3.5

Differentiate, and simplify $f(t) = (3t^2 - 2t)(5t^3 - 2t^2 + 1)^3$

$f'(t) = (6t-2)(5t^3-2t^2+1)^3 + (3t^2-2t)\left(3(5t^3-2t^2+1)^2(15t^2-4t)\right)$

$= (5t^3-2t^2+1)^2\left((6t-2)(5t^3-2t^2+1) + 3(3t^2-2t)(15t^2-4t)\right)$

Note: quiz or test - this is simple enough assignment - expand (the second factor) & collect like terms.

Example 2.3.6

Determine the derivative of $h(x) = \dfrac{3x-2}{5x^2+1}$

$\Rightarrow h(x) = (3x-2)(5x^2+1)^{-1}$ (change to a product fn)

$h'(x) = (3)(5x^2+1)^{-1} + (3x-2)\left(-(5x^2+1)^{-2}(10x)\right)$

$= \dfrac{3}{5x^2+1} - \dfrac{10x(3x-2)}{(5x^2+1)^2}$

$= \dfrac{3(5x^2+1) - 10x(3x-2)}{(5x^2+1)^2} = \dfrac{-15x^2+20x+3}{(5x^2+1)^2}$

use product

(expand & use power rule)

Example 2.3.7

Differentiate $s(t)=3t^2(2t-5)$

$s'(t) = (6t)(2t-5) + 3t^2(2)$

$= 6t(2t-5+t)$

$= 6t(3t-5)$

$s(t) = 6t^3 - 15t^2$

$s'(t) = 18t^2 - 30t$

$= 6t(3t-5)$

Example 2.3.8

Determine the slope of a tangent to $f(x)=2x^3(3x^2-5x+1)^4$ at $x=1$

$f'(x) = 6x^2(3x^2-5x+1)^4 + 2x^3\left(4(3x^2-5x+1)^3(6x-5)\right)$

$= 2x^2(3x^2-5x+1)^3\left(3(3x^2-5x+1) + x(6x-5)\right)$

$\therefore m_{tan} = f'(1) = (2)(-1)(-3+1)$

$= 4.$

Class/Homework for Section 2.3b

Pg. 90 – 91 #7 – 10, Pg. 92 #6 – 8