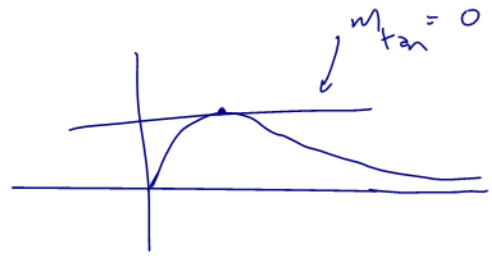
6. The concentration, c, of a drug in the blood t hours after the drug is taken orally is modelled by $c(t) = \dfrac{5t}{2t^2+7}$. How long after taking the drug does its concentration in the blood reach a maximum? (*From the Text: Pg. 98 #15*)

$$c'(t) = \frac{(5)(2t^2+7) - (5t)(4t)}{(2t^2+7)^2}$$



$$= \frac{-10t^2+35}{(2t^2+7)^2}$$

set $c'(t) = 0$

$$\Rightarrow \frac{-10t^2+35}{(2t^2+7)^2} = 0$$

$$\frac{a}{b} = 0 \Rightarrow a = 0$$

$$\Rightarrow -10t^2 + 35 = 0 \Rightarrow -10t^2 = -35$$

$$t^2 = \frac{35}{10} = 3.5$$

$$\Rightarrow t = \sqrt{3.5} = 1.87$$

2.5 Derivatives of Composite Functions

Thus far we have seen a number of simplifying rules for determining derivatives of various functions. We now turn to differentiating composite functions using the simplifying rule known as **The Chain Rule**.

Recall that given two functions $f(x)$ and $g(x)$, then the composition of $f(x)$ with $g(x)$ is defined to be:

$$\begin{aligned} F(x) &= (f \circ g)(x) \\ &= f(g(x)) \end{aligned}$$

Example 2.5.1

Given $F(x)=\sqrt{3x^2-5x+1}$ determine two functions $f(x)$ and $g(x)$ so that $f(g(x))=F(x)$. Also determine the composite function $G(x)=(g \circ f)(x)$.

$f(x) = \sqrt{x}$ $\quad g(x) = 3x^2 - 5x + 1$

$$\begin{aligned} G(x) = g(f(x)) &= 3(\sqrt{x})^2 - 5(\sqrt{x}) + 1 \\ &= 3x - 5\sqrt{x} + 1 \end{aligned}$$

The Chain Rule

Given two differentiable functions $f(x)$ and $g(x)$, then the composite function

$$F(x) = f(g(x))$$

is also differentiable, and

$$\frac{dF(x)}{dx} = \frac{df(g(x))}{dx} \cdot \frac{dg(x)}{dx}, \text{ or } F'(x) = f'(g(x)) \cdot g'(x)$$

In "words": The derivative of a composite f is

The derivative of the outer times the derivative of inner.

(LEAVE THE INNER ALONE)

Example 2.5.2

Differentiate:

a) $f(x)=\left(5x^2-7x+\sqrt{x}\right)^{97}$

$$f'(x)=97\left(5x^2-7x+\sqrt{x}\right)^{96}\cdot\left(10x-7+\frac{1}{2\sqrt{x}}\right)$$

b) $g(x)=\sqrt{3x^2-5x+1}$

$$g'(x)=\frac{1}{2\sqrt{3x^2-5x+1}}\cdot(6x-5)$$

$$=\frac{6x-5}{2\sqrt{3x^2-5x+1}}$$

c) $h(x)=\left(\dfrac{x+2}{3x-1}\right)^{12}$

inner is quotient! $\downarrow$ quotient rule

$$h'(x)=12\left(\frac{x+2}{3x-1}\right)^{11}\cdot\left(\frac{(1)(3x-1)-(x+2)(3)}{(3x-1)^2}\right)$$

$$=12\frac{(x+2)^{11}}{(3x-1)^{11}}\cdot\left(\frac{-7}{(3x-1)^2}\right)=\frac{-84(x+2)^{11}}{(3x-1)^{13}}$$

Example 2.5.3 (*A typical example using Leibniz Notation*)

Given that $V(x)=\underbrace{3x^2+2x-7}_{\frac{dV}{dx}}$, and that $x=\overbrace{3\sqrt{t}}^{\frac{dx}{dt}}$, determine:

a) $\dfrac{dV}{dt}$

b) $\dfrac{dV}{dt}$ when $t=9$

when $t=9$

$x=3\sqrt{9}=9$

$$\frac{dV}{dt}=\frac{dV}{dx}\cdot\frac{dx}{dt}$$

$$=(6x+2)\left(\frac{3}{2\sqrt{t}}\right)$$ //

b) $$\left.\frac{dV}{dt}\right|_{\substack{t=9\\x=9}}=\left(6(9)+2\right)\left(\frac{3}{2\sqrt{9}}\right)$$

$$=28$$

Traditional technique

(make a composite fn)

$$V(t)=3(3\sqrt{t})^2+2(3\sqrt{t})-7$$

$$=27t+6\sqrt{t}-7$$

$$\frac{dV}{dt}=27+\frac{3}{\sqrt{t}}$$

b) $$\left.\frac{dV}{dt}\right|_{t=9}=27+\frac{3}{\sqrt{9}}$$

$$=28$$

Example 2.5.4

Determine the equation of the tangent to $y=(3x+2)^{\frac{1}{3}}$ at $x=2$ $\Rightarrow$ Point is $(2, y_1|_{x=2})$

$y = m_{tan} x + b$ $= (2,2)$

$$y' = \frac{1}{3}(3x+2)^{-2/3} \cdot 3 = \frac{\cancel{3}\ 1}{\cancel{3}(3x+2)^{2/3}}$$

$$= \frac{1}{(3x+2)^{2/3}}$$

$$m_{tan} = y'\Big|_{x=2} = \frac{1}{(3(2)+2)^{2/3}} = \frac{1}{8^{2/3}} = \frac{1}{4}$$

$\therefore$ we have $y = \frac{1}{4}x + b$ use $(2,2)$

$$2 = \frac{1}{4}(2) + b$$

$$b = 3/2$$

$$\boxed{y = \frac{1}{4}x + \frac{3}{2}.}$$

Class/Homework for Section 2.5

Pg. 105 – 106 #1 – 3, 4, 5, 7 – 10, 13, 15, 16