A∞Ω
MATH@TD

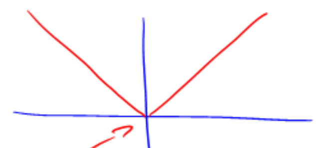
MCV4U Practice

1. Determine $f'(2)$ for $f(x) = x^2 + 4x - 1$. ____
   a. 7
   b. 8
   c. 11
   d. 12

$f'(x) = 2x + 4$
$\Rightarrow f'(2) = 8$

2. All but one of the functions is differentiable for all real values of x. Which function is not differentiable for at least one real value of x? ____
   a. $f(x) = x^2 + 1$
   b. $g(x) = \dfrac{1}{x^2 + 1}$
   c. $h(x) = |x|$
   d. $j(x) = x^3 - 3x$

$h(x) = |x|$ has a "cusp" or 'sharp point' at $x = 0$ $\therefore$ not diff'ble at $x = 0$



not diff'ble at $x = 0$

3. Determine the derivative $\dfrac{dy}{dx}$ for $y = 2x^3 - 3x + 1$. ____
   a. $6x^2 - 3$
   b. $6x^2 - 3x$
   c. $3x^2 - 3$
   d. $x^2 - 3$

$\dfrac{dy}{dx} = 6x^2 - 3$
(power rule)

4. Determine $\dfrac{dy}{dx}$ for $y = \dfrac{x^2 - 4}{x^2 + 4}$ when $x = 1$. ____
   a. $-\dfrac{16}{25}$
   b. $\dfrac{4}{25}$
   c. $\dfrac{16}{25}$
   d. 1

$$y' = \frac{2x(x^2+4) - (x^2-4)(2x)}{(x^2+4)^2}$$
(quotient rule)

$$\therefore y'\Big|_{x=1} = \frac{2(5) - (-3)(2)}{(5)^2} = \frac{16}{25}$$

could simplify by 'collecting like terms' in the numerator - but no need here since we are calculating a value.

5. The position s, in metres, of an object moving in a straight line is given by $s(t) = 5t(t-2)^2$, where t is the time in seconds. Determine the velocity of the object at time $t = 1$. _____

a. 15 m/s
b. 5 m/s
c. 0 m/s
d. –5 m/s

$$v(t) = s'(t) = 5(t-2)^2 + 5t(2(t-2))$$ (product rule + chain rule)

$$\Rightarrow v(1) = 5(-1)^2 + 5(1)(2(-1))$$

$$= -5 \text{ m/sec}$$

6. An initial population, p, of 1500 bacteria grows in number according to the equation $p(t) = 1500\left(1 + \frac{5t}{t^2 + 30}\right)$, where t is in hours. Determine the rate at which the population is growing after 3 h. _____

a. 0.069 bacteria/h
b. 104 bacteria/h
c. 281 bacteria/h
d. 4038 bacteria/h

$$p(t) = 1500 + \frac{7500t}{t^2+30} \Rightarrow p'(t) = \frac{7500(t^2+30) - 7500t(2t)}{(t^2+30)^2}$$

(simplifying $p(t)$)

(quotient rule)

$$\Rightarrow p'(3) = \frac{7500(39) - 7500(3)(6)}{(39)^2}$$

$$\doteq 103.55 \text{ bacteria/hr.}$$

7. For which value(s) of x is the tangent to $f(x) = \dfrac{x^2+3}{x+1}$ horizontal?

– tangent has zero slope

$\Rightarrow m_{tan} = f'(a) = 0$

need to differentiate

a. $x = 1$
b. $x = -3, 1$
c. $x = -1, 3$
d. $x = 3$

$$f'(x) = \frac{2x(x+1) - (x^2+3)(1)}{(x+1)^2}$$ (quotient rule)

set to zero

$$\Rightarrow \frac{2x^2 + 2x - x^2 - 3}{(x+1)^2} = 0$$ $\left(\frac{a}{b} = 0 \Rightarrow a = 0!\right)$

$$\Rightarrow x^2 + 2x - 3 = 0$$

$$\Rightarrow (x+3)(x-1) = 0 \Rightarrow x = -3 \text{ OR } 1$$

8. Determine the value of k for which $f'(3) = 2$, if $f(x) = \dfrac{x+k}{x-1}$.

a. -9 c. 5
b. -5 d. 9

$$f'(x) = \frac{(1)(x-1) - (x+k)(1)}{(x-1)^2}$$ (quotient rule)

$$\Rightarrow f'(x) = \frac{-1-k}{(x-1)^2}$$ using $f'(\overset{x\text{-value}}{3}) = \underset{}{2}$ ← derivative value (slope of tangent)

$$\Rightarrow 2 = \frac{-1-k}{(2)^2} \Rightarrow 8 = -1-k$$
$$\Rightarrow k = -9$$

9. If $f(x) = \sqrt{x^2 - 1}$ and $g(x) = x + 1$, which expression is equal to $f(g(x))$?

a. $1 + \sqrt{x^2 - 1}$ c. $(x+1)^2 - 1$
b. $\sqrt{x^2 + 2x}$ d. $\sqrt{x^2 + x - 1}$

← composition of fns. with "g inside f".

$$f(g(x)) = \sqrt{(x+1)^2 - 1}$$
$$= \sqrt{x^2 + 2x + 1 - 1}$$ (expanding and collecting like terms)
$$= \sqrt{x^2 + 2x}$$

10. Determine the slope of the tangent to the curve $y = (2x - 3x^2)^2$ at (1, 1).

a. -16 c. -2
b. -8 d. 8

$$y' = 2(2x - 3x^2)(2 - 6x)$$ (chain rule)

$$m_{tan} = y'\Big|_{x=1} = 2(2-3)(2-6)$$
$$= +8$$

11. a. Use the formal definition of the derivative to determine $f'(x)$ for $f(x) = \sqrt{x-1}$.
b. Determine the slope of the tangent to $f(x)$ at (10, 3).

(a question like this will be on your test)

a)

$$f'(x) = \lim_{h\to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

("formal defn" is the limit defn)

$$= \lim_{h\to 0}\left(\frac{\sqrt{x+h-1}-\sqrt{x-1}}{h}\right)$$

(conjugate)

$$= \lim_{h\to 0}\left(\frac{\sqrt{x+h-1}-\sqrt{x-1}}{h}\cdot\frac{\sqrt{x+h-1}+\sqrt{x-1}}{\sqrt{x+h-1}+\sqrt{x-1}}\right)$$

$$= \lim_{h\to 0}\left(\frac{(x+h-1)-(x-1)}{h\left(\sqrt{x+h-1}+\sqrt{x-1}\right)}\right)$$

$$= \lim_{h\to 0}\left(\frac{\cancel{h}}{\cancel{h}\left(\sqrt{x+h-1}+\sqrt{x-1}\right)}\right)$$

$$= \frac{1}{2\sqrt{x-1}}$$

b) $m_{tan} = f'(10) = \dfrac{1}{2\sqrt{9}} = \dfrac{1}{6}$

(I could also ask for the equation of the tangent!)

in case you care, the eqn of the tangent @ (10,3) is $y-3 = \frac{1}{6}(x-10)$

$\Rightarrow \boxed{x-6y+8=0}$

12. Determine the values of a, b, and c for $f(x) = ax^2 + bx + c$ so that $f'(x) = 6x - 3$ and $f(2) = -1$.

$f(2) = -1$

$\Rightarrow a(2)^2 + b(2) + c = -1$

$\Rightarrow 4a + 2b + c = -1$ ①

$f'(x) = 2ax + b$

$\Rightarrow 2ax + b = 6x - 3$ (compare coefficients)

$\Rightarrow 2a = 6$ and $\boxed{b = -3}$

$\Rightarrow \boxed{a = 3}$

Sub into ①

$\Rightarrow 4(3) + 2(-3) + c = -1$

$\Rightarrow \boxed{c = -7}$

13. The population of a rabbits in a controlled system can be described by $P(x) = -x^2 + 18x + 19$, where x is the number of years after the population was first tracked.

a. What is the meaning of $P'(x)$ for this scenario?
b. $P'(11) = -4$. Explain what this means.

a) $P'(x)$ is the rate that the rabbit population is changing at any time, x

b) At 11 years, the rabbit population is decreasing at a rate of 4 "units of rabbits"/year. (eg 4000 rabbits/yr or 40 rabbits/yr or ...)

the negative tells us this

14. a. Determine the derivative of $f(x) = (x+1)^2(3x^2-5)^4$. Write your answer in simplified factored form.
b. Determine the value(s) of x for which the graph of $f(x)$ has a horizontal tangent.

a) $f'(x) = 2(x+1)(3x^2-5)^4 + (x+1)^2\left(4(3x^2-5)^3(6x)\right)$

$= 2(x+1)(3x^2-5)^3\left((3x^2-5) + 12x(x+1)\right)$

$= 2(x+1)(3x^2-5)^3(15x^2+12x-5)$

b) (next page)

b) we want $f'(x) = 0$

$\Rightarrow 2(x+1)(3x^2-5)^2(15x^2+12x-5) = 0$

$\Rightarrow 2(x+1) = 0$ OR $(3x^2-5)^2 = 0$ OR $(15x^2+12x-5) = 0$

$\Rightarrow \boxed{x = -1}$

$\Rightarrow 3x^2 - 5 = 0$

$\Rightarrow \boxed{x = \pm\sqrt{\frac{5}{3}}}$

Quadratic Formula

$x = \frac{-12 \pm \sqrt{12^2 - 4(15)(-5)}}{2(15)}$

$\Rightarrow x = \frac{-12 \pm \sqrt{144 + 300}}{30}$

$\Rightarrow \boxed{x = 0.30}$ OR $\boxed{x = -1.10}$

(this question is a bit mean, but it's a good question overall)

15. a. Determine the point $(a, f(a))$ for which $f'(a) = a$, given that $f(x) = -x^2 + 3x - 7$.
b. Write the equation of the tangent to $f(x)$ at the point found in part a.

a) $f'(x) = -2x + 3$

we want $f'(a) = a$

$\Rightarrow a = -2(a) + 3$

$\Rightarrow 3a = 3 \Rightarrow a = 1$

b) Point $(1, f(1)) = (1, -5)$

$m_{tan} = f'(1) = -2(1) + 3 = 1$

eqn of tangent

$y - y_1 = m(x - x_1)$

$\Rightarrow y + 5 = (1)(x-1)$

$\Rightarrow \boxed{x - y - 5 = 0}$

16. a. Determine $f'(x)$ if $f(x) = (8x^2 + x)^3$. Write your answer in simplified form.

b. Determine $f'\left(\frac{1}{2}\right)$.

a) $f'(x) = 3(8x^2 + x)^2(16x + 1)$ (chain rule)

(this is factored form already, so no simplification needed)

b) $f'(\tfrac{1}{2}) = 3\left(8(\tfrac{1}{2})^2 + \tfrac{1}{2}\right)^2\left(16(\tfrac{1}{2}) + 1\right)$

$= 3(2.5)^2(9)$

$= 168.75$

17. a. Determine $\frac{dy}{dx}$ if $y = \sqrt{(3x^2 + 2)^3}$. Write your answer in simplified form.

b. State any values of x for which the function is not differentiable.

a) $y = (3x^2 + 2)^{3/2}$ (rewriting to make the 'power' easier to deal with)

(use the chain rule)

$y' = \frac{3}{2}(3x^2 + 2)^{\frac{1}{2}}(6x)$ bring out front to simplify $6 \times \frac{3}{2} = 9!$

$= 9x(3x^2 + 2)^{\frac{1}{2}}$

b) y is differentiable everywhere on its domain. The domain of the fn is all real numbers.

18. a. Differentiate $f(x) = \dfrac{(x+2)^3}{(x+3)(2x-1)}$. Simplify your answer.

b. State which rules of differentiation you used.

a) $f'(x) = \dfrac{\underbrace{3(x+2)^2}_{\text{chain rule}}\left((x+3)(2x-1)\right) - (x+2)^3\left((1)(2x-1) + (x+3)(2)\right)}{\left((x+3)(2x-1)\right)^2}$

product rule for "derivative of the bottom"

quotient rule overall

$= \dfrac{3(x+2)^2(x+3)(2x-1) - (x+2)^3(4x+5)}{(x+3)^2(2x-1)^2}$

$(x+2)^2$ is a common factor on top.

next page for simplification

$$= \frac{(x+2)^2\left(3(x+3)(2x-1) - (x+2)(4x+5)\right)}{(x+3)^2(2x-1)^2}$$

expand & collect like terms

$$= \frac{(x+2)^2\left(3(2x^2+5x-3) - (4x^2+13x+10)\right)}{(x+3)^2(2x-1)^2}$$

$$= \frac{(x+2)^2\left(2x^2+2x-19\right)}{(x+3)^2(2x-1)^2}$$

does not factor further

So – we're done!

b) We use the quotient rule, the chain rule and the product rule.