Chapter 2: Derivatives – Test (From a Previous Year)

Multiple Choice

Circle the choice that best completes the statement or answers the question. Also write the letter corresponding to your choice on the appropriate line.

K: ___ / ***4*** ***A:*** ___ / ***2*** ***T:*** ___ / ***4***

1. Determine $f'(2)$ for $f(x) = x^2 + 4x - 1$. _____
   a. 7
   b. 8
   c. 11
   d. 12
2. All but one of the functions is differentiable for all real values of *x*. Which function is not differentiable for at least one real value of *x*? _____
   a. $f(x) = x^2 + 1$
   b. $g(x) = \dfrac{1}{x^2 + 1}$
   c. $h(x) = |x|$
   d. $j(x) = x^3 - 3x$
3. Determine the derivative $\dfrac{dy}{dx}$ for $y = 2x^3 - 3x + 1$. _____
   a. $6x^2 - 3$
   b. $6x^2 - 3x$
   c. $3x^2 - 3$
   d. $x^2 - 3$
4. Determine $\dfrac{dy}{dx}$ for $y = \dfrac{x^2 - 4}{x^2 + 4}$ when $x = 1$. _____
   a. $-\dfrac{16}{25}$
   b. $\dfrac{4}{25}$
   c. $\dfrac{16}{25}$
   d. 1
5. The position *s*, in metres, of an object moving in a straight line is given by $s(t) = 5t(t-2)^2$, where *t* is the time in seconds. Determine the velocity of the object at time $t = 1$. _____
   a. 15 m/s
   b. 5 m/s
   c. 0 m/s
   d. –5 m/s
6. An initial population, *p*, of 1500 bacteria grows in number according to the equation $p(t) = 1500\left(1 + \dfrac{5t}{t^2 + 30}\right)$, where *t* is in hours. Determine the rate at which the population is growing after 3 h. _____
   a. 0.069 bacteria/h
   b. 104 bacteria/h
   c. 281 bacteria/h
   d. 4038 bacteria/h
7. For which value(s) of *x* is the tangent to $f(x) = \dfrac{x^2 + 3}{x + 1}$ horizontal? _____
   a. $x = 1$
   b. $x = -3, 1$
   c. $x = -1, 3$
   d. $x = 3$
8. Determine the value of *k* for which $f'(3) = 2$, if $f(x) = \dfrac{x + k}{x - 1}$. _____
   a. –9
   b. –5
   c. 5
   d. 9

9. If $f(x) = \sqrt{x^2 - 1}$ and $g(x) = x + 1$, which expression is equal to $f(g(x))$? _____

   a. $1 + \sqrt{x^2 - 1}$
   c. $(x+1)^2 - 1$
   b. $\sqrt{x^2 + 2x}$
   d. $\sqrt{x^2 + x - 1}$

10. Determine the slope of the tangent to the curve $y = (2x - 3x^2)^2$ at (1, 1). _____

   a. –16
   c. –2
   b. –8
   d. 8

Full Solutions: Place your solutions to the following problems in the appropriate places on the Answer Sheets. *Note: a Communication mark, out of 10, will be given for how well you present your solutions, mathematically.*

11. a. Use the formal definition of the derivative to determine $f'(x)$ for $f(x) = \sqrt{x - 1}$. **(A: 4)**
    b. Determine the slope of the tangent to $f(x)$ at (10, 3). **(K: 1)**

12. Determine the values of a, b, and c for $f(x) = ax^2 + bx + c$ so that $f'(x) = 6x - 3$ and $f(2) = -1$. **(T: 4)**

13. The population of a rabbits in a controlled system can be described by $P(x) = -x^2 + 18x + 19$, where x is the number of years after the population was first tracked. **(K: 4)**

    a. What is the meaning of $P'(x)$ for this scenario?
    b. $P'(11) = -4$. Explain what this means.

14. a. Determine the derivative of $f(x) = (x + 1)^2(3x^2 - 5)^4$. Write your answer in simplified factored form. **(T: 4)**
    b. Determine the value(s) of x for which the graph of $f(x)$ has a horizontal tangent. **(A: 2)**

15. a. Determine the point $(a, f(a))$ for which $f'(a) = a$, given that $f(x) = -x^2 + 3x - 7$. **(T: 3)**
    b. Write the equation of the tangent to $f(x)$ at the point found in part a. **(K: 2)**

16. a. Determine $f'(x)$ if $f(x) = (8x^2 + x)^3$. Write your answer in simplified form. **(T: 4)**
    b. Determine $f'\left(\frac{1}{2}\right)$. **(K: 1)**

17. a. Determine $\frac{dy}{dx}$ if $y = \sqrt{\left(3x^2 + 2\right)^3}$. Write your answer in simplified form. **(T: 4)**
    b. State any values of x for which the function is not differentiable. **(K: 2)**

18. a. Differentiate $f(x) = \frac{(x+2)^3}{(x+3)(2x-1)}$. Simplify your answer. **(T: 4)**
    b. State which rules of differentiation you used. **(K: 1)**