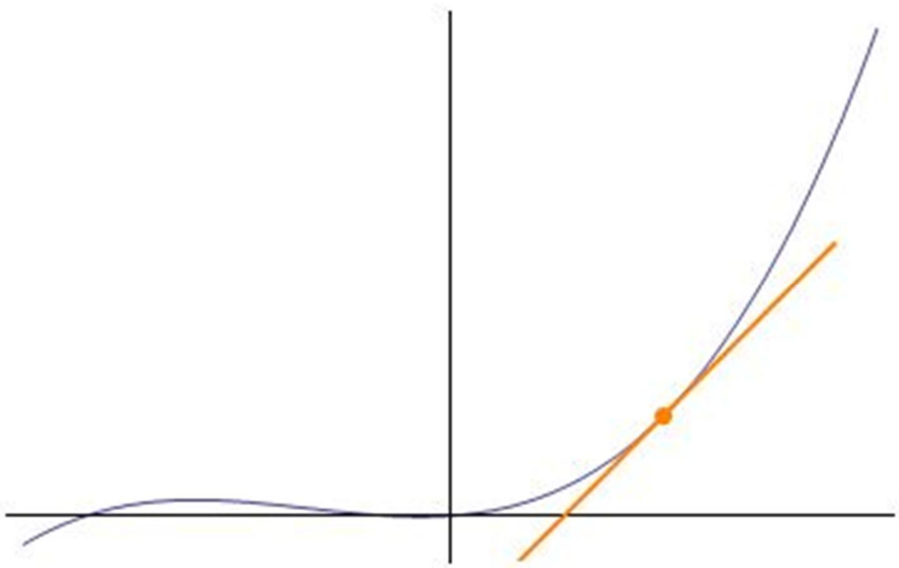
CALCULUS

Chapter 3 –Applications of the Derivative

(*Material adapted from Chapter 3 of your text*)



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MATH@TD

Chapter 3 – Applications of the Derivative

Contents with suggested problems from the Nelson Textbook (Chapter 3)

3.1 Higher Order Derivatives: Velocity and Acceleration

Higher Order Derivatives

Recall that given some function $f(x)$ we can find the derivative $f'(x)$ which is itself a function. Thus, we should be able to find the "**derivative of the derivative**". This is the essence of **higher order derivatives**.

Example 3.1.1

Consider the function:

$$f(x) = 3x^4 - 2x^2 + 1$$

Question

Given a polynomial function of degree *n*, what is the maximum number of derivatives that can be calculated?

(∞'ly many, but "pointless" after $(n+1)^{th}$ derivative

$$\Rightarrow \frac{df(x)}{dx} = 12x^3 - 4x$$

Taking another derivative (or the deriv. of deriv.)

2ND derivative

$$\frac{d}{dx}\left(\frac{df}{dx}\right) = \frac{d^{2}f}{dx^{2}} = 36x^2 - 4 \qquad (f''(x))$$

3RD derivative (in prime notation too)

$$\frac{d^3 f(x)}{dx^3} = 72x \qquad \left(\text{in prime: } f'''(x)\right)$$

4th derivative

4th deriv.

$$\frac{d^4 f(x)}{dx^4} = 72 \qquad \left(\text{in prime: } f^{(4)}(x)\right)$$

5th deriv

$$\frac{d^5 f(x)}{dx^5} = 0 \qquad \left(\text{in prime } f^{(5)}(x)\right)$$

Position, Velocity and Acceleration

We will consider the motion of a particle in a straight line for our considerations of velocity and acceleration. One thing will be necessary. **We must define a positive direction of motion**, and interpret our results in light of that definition.

Example 3.1.2

Given that an object is moving in a straight line, and that the object's position is defined/modelled by

$$s(t) = t^3 - 6t^2, \quad t \ge 0$$

Determine:

a) $v(t)$ $\Rightarrow$ $\dfrac{\text{rate of change } s(t)}{\text{rate of change of } t} = \dfrac{ds}{dt} = s'(t)$

b) $a(t)$ $= \dfrac{\text{rate of change of vel}}{\text{change of time}} = v'(t) = s''(t)$

c) in what direction the object is moving at $t = 1$sec, $t = 3$ sec, $t = 5$ seconds.

Assume that moving to the right is motion in the positive direction.

d) the object's acceleration at the same times as in part c).

e) when the object is at rest.

f) when the acceleration is zero

g) draw a **position diagram** describing the motion of the object.

$s(t) = t^3 - 6t^2$

a) $v(t) = s'(t) = 3t^2 - 12t$

b) $a(t) = v'(t) = 6t - 12$

$(= s''(t))$

c) velocity tells us the direction of motion : Positive velocity is motion to right

Neg. " " " " left

We need to calculate

$v(1) = -9$ m/sec   moving left at $t = 1$ sec

$v(3) = -9$ m/sec   moving left at $t = 3$ sec

$v(5) = +15$ m/sec   moving right at $t = 5$ sec

d) We want $a(1) = 6(1) - 12 = -6 \ m/sec^2$

(velocity is negative $\Rightarrow$ motion is left
accl^n is left
$\Rightarrow$ particle is "speeding up" to left)

$a(3) = +6 \ m/sec^2$ (particle is "slowing down to left")

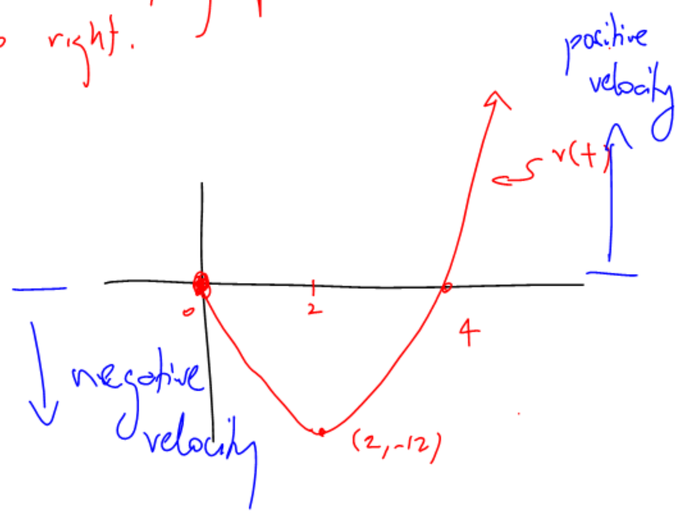
$a(5) = +18 \ m/sec$ particle is speeding up to right.

e) We need $v(t) = 0$

$\Rightarrow 3t^2 - 12t = 0$

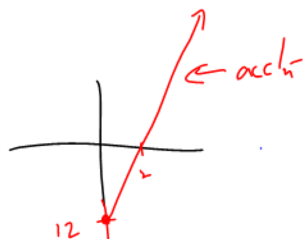
$\Rightarrow 3t(t-4) = 0$

$\Rightarrow t = 0$ or $t = 4$



f) $a(t) = 0$

$\Rightarrow 6t - 12 = 0 \Rightarrow t = 2$

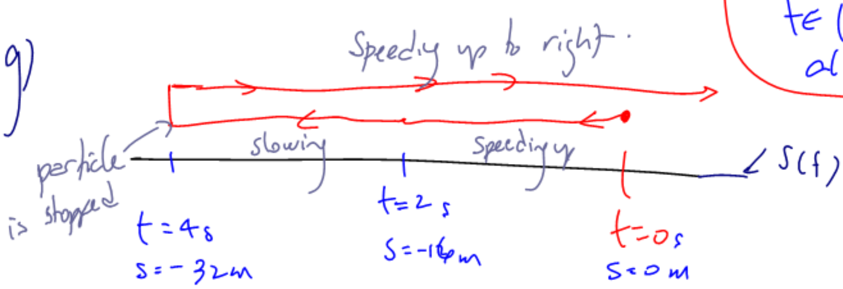


By the sketch for $t \in (0,4)$
$v(t) < 0$
$\Rightarrow$ leftward motion

for $t > 4$, $v(t) > 0$
$\Rightarrow$ rightward motion

$t \in (0,2)$
$a(t) < 0$
$t \in (2, \infty)$
$a(t) > 0$

g)



We need position values for $t = 0, 2, 4$

stationary points

$s(0) = 0$	$v(0) = 0$
$s(2) = -16 \ m$	$v(2) = -12$
$s(4) = -32 \ m$	$v(4) = 0$

Class/Homework for Section 3.1

Pg. 127 - 129 #2 – 5, 8, 10, 12 – 16, Read Ex's 2 and 4