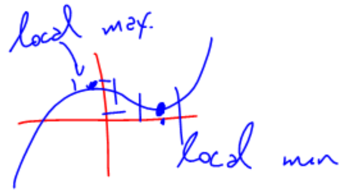


3.2 Extreme Values

The **Maximum** or **Minimum** values which some function may have are called **Extreme Values**.

Definition 3.2.1

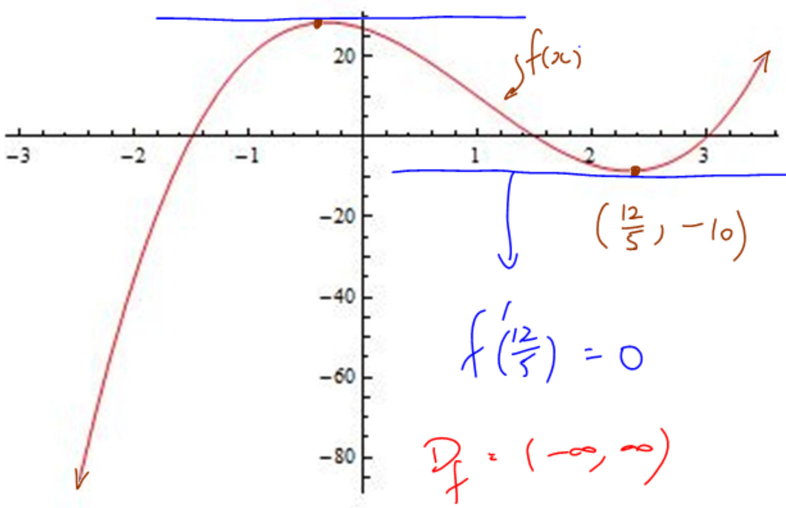
If a differentiable function, $f(x)$, has a **local extremum** at a domain value $x = c$, then

$$f'(c) = 0$$

Pictures

$f'(-2/5) = 0$

$(-\frac{2}{5}, 27)$



$(\frac{12}{5}, -10)$

$f'(\frac{12}{5}) = 0$

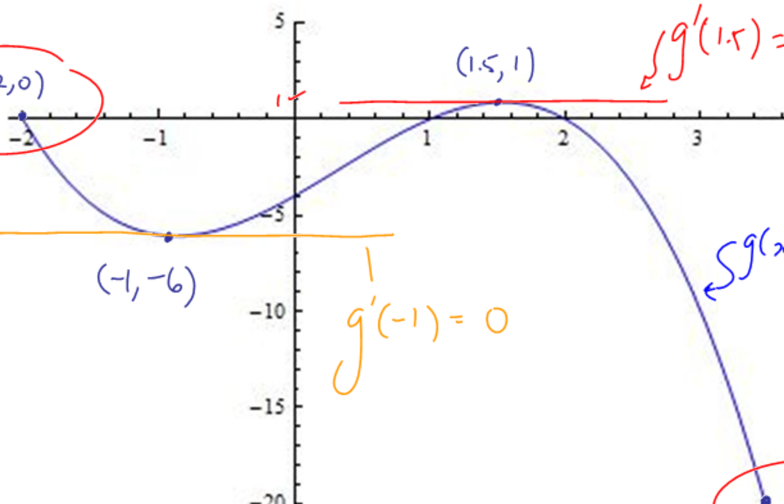
$D_f = (-\infty, \infty)$

$f(x)$ has a LOCAL MAX OF 27 at $x = -2/5$ and LOCAL MIN of -10 at $x = \frac{12}{5}$

Note: $f(x)$ does NOT have an absolute max or min.

$(-2, 0)$ endpoint

$(1.5, 1)$ $g'(1.5) = 0$



$(-1, -6)$

$g'(-1) = 0$

$(3.5, -20)$ endpoint

$g(x)$ has a local max at $x = 1.5$ of 1

BUT the value $g(1.5) = 1$ is ALSO an absolute max

local min of -6 at $x = -1$

$D_g = [-2, 3.5]$

$g(x)$ ALSO has an absolute min of -20 at $x = 3.5$

Definition 3.2.2

Given a differentiable function, $f(x)$, we call any point $(c, f(c))$ a **critical point** of $f(x)$ whenever $f'(c) = 0$ (we call such domain values, $x=c$, critical values)

The Extreme Value Theorem

(E.V.T.)

Given a differentiable function, $f(x)$, defined on a **closed interval** $x \in [a,b]$, **then** $f(x)$ is **guaranteed** an **absolute/global maximum**, and an **absolute/global minimum**. These **absolute extrema** occur at the endpoints of the interval, or at points $(c, f(c))$, inside the interval, where $a < c < b$, and where $f'(c) = 0$.

(*No Proof given*)

The Algorithm for Finding Absolute/Global Extreme Values

> Note: *In this explanation we assume that the functions we are working with are differentiable on the given closed interval.*

1) Differentiate $f(x)$ (defined on the closed interval $x \in [a,b]$), and find all domain values $x = c$ $(c \in (a,b))$ where $f'(c) = 0$. Such **domain values** are called **critical values**.

2) Test all critical values, $x = c$, and the domain values of the endpoints, $x = a$, and $x = b$, in the function $f(x)$ to calculate the absolute/global max and min values.

Example 3.2.1

Determine the absolute extrema of:

1) $f(x) = \dfrac{1}{x}$ on $x \in [-1, 2]$

2) $g(x) = x^3 - 3x^2 - 9x + 2$ on $x \in [-2, 2]$

we are not guaranteed any absolute extrema

1) Note: the E.V.T. does not apply to $f(x) = \frac{1}{x}$ on $[-1,2]$ since $f(0)$ is undefined and $0 \in [-1,2]$. $\therefore f(x)$ is not differentiable on $[-1,2]$

2) $g(x) = x^3 - 3x^2 - 9x + 2$ on $[-2, 2]$

$g'(x) = 3x^2 - 6x - 9$

C.V. $\Rightarrow$ $g'(x) = 0$

$\Rightarrow 3x^2 - 6x - 9 = 0$

$\div 3 \quad x^2 - 2x - 3 = 0$

$\Rightarrow (x-3)(x+1) = 0$

$\therefore$ The critical values are $x = 3$, $x = -1$

Note: $g(x)$ is a polynomial. $\therefore g(x)$ is diff'ble $\therefore$ The EVT does apply.

Note: $3 \notin [-2, 2]$ $\therefore$ don't consider it

Test $x = -1$ cv

$x = -2$, $x = 2$ } endpoints

$g(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 2 = 7$

$g(-2) = 0$

$g(2) = -20$

$\therefore g(x)$ has an absolute max of 7 at $x = -1$ and an absolute min of -20 at $x = 2$

Class/Homework for Section 3.2

Pg. 135 – 138 #1 – 4, 6 – 13