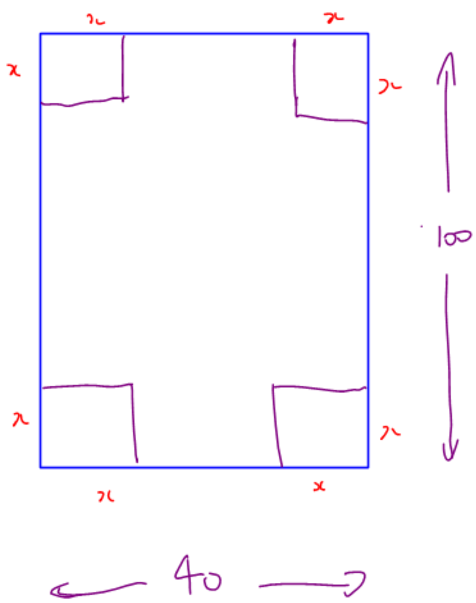
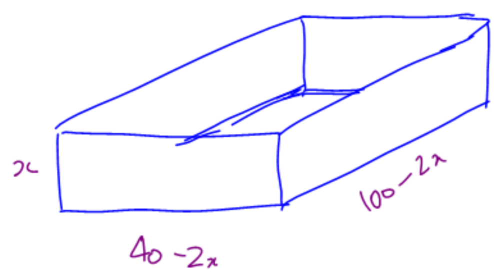
Hwk Check – §3.3

4. A rectangular piece of cardboard, 100 cm by 40 cm, is going to be used to make a rectangular box with an open top by cutting congruent squares from the corners. Calculate the dimensions (to one decimal place) for a box with the largest volume.

Pictures





$V(x) = x(40-2x)(100-2x)$

$0 \leq x \leq 20$

$V(x) = 4x^3 - 280x^2 + 4000x$

$\therefore V'(x) = 12x^2 - 560x + 4000$

C.V. $\Rightarrow$ $V'(x) = 0$

$\Rightarrow 12x^2 - 560x + 4000 = 0$

$\div 4$ $\quad 3x^2 - 140x + 1000 = 0$

by Q.F. $\quad x = \dfrac{140 \pm \sqrt{(-140)^2 - 4(3)(1000)}}{6}$

$= \dfrac{140 \pm 87.2}{6}$

$\therefore x = 37.8$ or $x = 8.8$

inadmissable

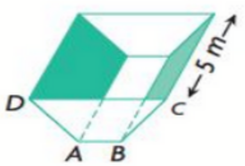
Test $x = 0,\ 8.8,\ 20$ in V (but $V(0) = 0$, $V(20) = 0$)

$V(8.8) = 16{,}242.7 \text{ cm}^3$ which is the max

$\therefore$ The dimensions which maximize the volume are

$h = 8.8 \text{ cm}$, $w = 40 - 2(8.8) = 22.4 \text{ cm}$, $l = 100 - 2(8.8) = 82.4 \text{ cm}$.

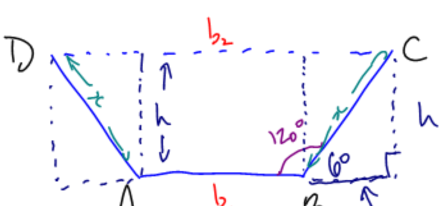
13. a. An isosceles trapezoidal drainage gutter is to be made so that the angles at A and B in the cross-section $ABCD$ are each 120°. If the 5 m long sheet of metal that has to be bent to form the open-topped gutter and the width of the sheet of metal is 60 cm, then determine the dimensions so that the cross-sectional area will be a maximum.



$0 \le x \le 30$

b. Calculate the maximum volume of water that can be held by this gutter.

Picture



$x \cdot \cos(60) = \frac{x}{2}$

$2x + b_1 = 60$

$\boxed{b_1 = 60 - 2x}$ $(*)$

$\sin(60) = \frac{h}{x}$

$\Rightarrow h = x \cdot \sin(60)$

$\boxed{h = \frac{\sqrt{3}\,x}{2}}$ $(**)$

$b_2 = b_1 + 2\left(\frac{x}{2}\right)$

$= b_1 + x$

$= 60 - 2x + x$

$\boxed{b_2 = 60 - x}$ $(***)$

Subbing into A

$A = \frac{1}{2}(b_1 + b_2)h$

need a single variable!

$A = \frac{1}{2}\left(60 - 2x + 60 - x\right)\left(\frac{\sqrt{3}\,x}{2}\right)$

$= \frac{1}{2}(120 - 3x)\left(\frac{\sqrt{3}}{2}x\right)$

$= 30\sqrt{3}\,x - \frac{3\sqrt{3}}{4}x^2$

$A'(x) = 30\sqrt{3} - \frac{3\sqrt{3}}{2}x$

set to zero for C.V.

$\Rightarrow 30\sqrt{3} - \frac{3\sqrt{3}}{2}x = 0$

$\div 3\sqrt{3}$ $\quad 10 - \frac{1}{2}x = 0$

$\therefore x = 20$

Test $x = 0, 20, 30$

$A(0) = 0$

$A(20) = 519.6 \text{ cm}^2$

$A(30) = 259.8 \text{ cm}^2$

$\therefore$ The dimensions are $b_1 = 60 - 2(20)$ $= 20 \text{ cm}$, $b_2 = 60 - 20$ $= 40 \text{ cm}$

$h = \frac{\sqrt{3}(20)}{2} = 10\sqrt{3} \text{ cm}$

to maximize area

b) $V = (\text{cross sectional area})(\text{length})$

$= (519.6 \text{ cm}^2)(500 \text{ cm})$ units must agree!

$= 259{,}800 \text{ cm}^3$

$= (0.259 \text{ m}^3)$