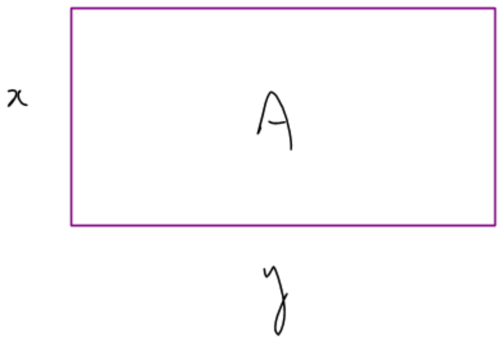
Hwk Check §3.3

6. What are the dimensions of a rectangle with an area of 64 m^2 and the smallest possible perimeter? (Note: ~~x ≥ 1~~ the smallest dimension is 1 m)

Picture



$A = xy$

$64 = xy$

$y = \frac{64}{x}$ (*) Sub into P

$1 \leq x \leq 64$

Test $x = 1, 8, 64$ in $P(x)$

$P(1) = 130$

$\boxed{P(8) = 32}$

$P(64) = 130$

$P = 2x + 2y$

$\Rightarrow P = 2x + 2\left(\frac{64}{x}\right)$

$P(x) = 2x + \frac{128}{x}$

$P'(x) = 2 - \frac{128}{x^2}$

Set to zero for C.V.

$\Rightarrow 2 - \frac{128}{x^2} = 0$

$2x^2 = 128$

$x^2 = 64$

$\Rightarrow x = \pm 8$ (but -8 is inadmissable)

$\Rightarrow x = 8$

$\therefore$ The dimensions which give the minimum perimeter of 32 cm are

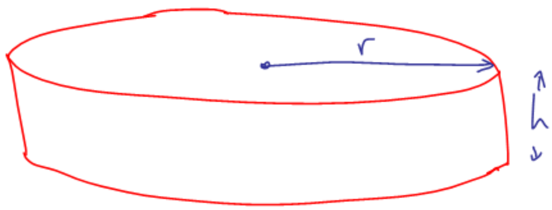
$w = 8$ cm and $l = \frac{64}{8}$ (by *)

$= 8$ cm

11. A cylindrical-shaped tin can must have a capacity of 1000 cm^3.

a. Determine the dimensions that require the minimum amount of tin for the can. (Assume no waste material.) According to the marketing department, the smallest can that the market will accept has a diameter of 6 cm and a height of 4 cm.

Picture



$V = \pi r^2 h$

$1000 = \pi r^2 h$

$\Rightarrow h = \dfrac{1000}{\pi r^2}$ $(*)$

$3 \le r \le \sqrt{\dfrac{250}{\pi}}$

Sub $(*)$ in SA

side, top + bottom.

$SA = 2\pi r h + 2\pi r^2$

$\Rightarrow SA = 2\pi r\left(\dfrac{1000}{\pi r^2}\right) + 2\pi r^2$.

$SA(r) = \dfrac{2000}{r} + 2\pi r^2$.

$SA'(r) = -\dfrac{2000}{r^2} + 4\pi r$

set to zero for C.V.

$\Rightarrow 4\pi r - \dfrac{2000}{r^2} = 0$.

$4\pi r^3 = 2000$

$\Rightarrow r = \left(\dfrac{2000}{4\pi}\right)^{\frac{1}{3}}$

$= 5.42$ cm

Test $r = 3$

$r = 5.42$ in SA

$r = \sqrt{\dfrac{250}{\pi}}$

$SA(3) = 723.2$ cm^2

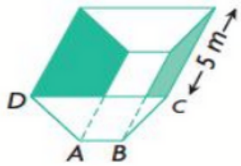
$SA(5.42) = 553.58$ cm.

$SA\left(\sqrt{\dfrac{250}{\pi}}\right) = 556.04$ cm

$\therefore$ The dimensions which minimize the SA are

$r = 5.42$ cm, $h = \dfrac{1000}{\pi(5.42)^2} = 10.84$ cm (by $*$)

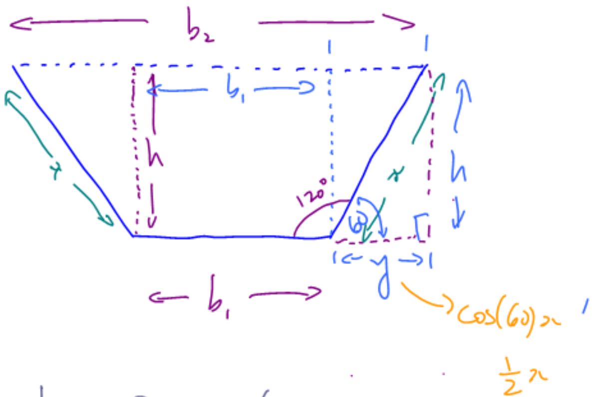
13. a. An isosceles trapezoidal drainage gutter is to be made so that the angles at A and B in the cross-section $ABCD$ are each 120°. If the 5 m long sheet of metal that has to be bent to form the open-topped gutter and the width of the sheet of metal is 60 cm, then determine the dimensions so that the cross-sectional area will be a maximum.



b. Calculate the maximum volume of water that can be held by this gutter.

$0 \le x \le 30$

Picture



$b_1 + 2x = 60$

$\Rightarrow \boxed{b_1 = 60 - 2x}$ (*)

$\sin(60) = \dfrac{h}{x}$

$\Rightarrow h = x \cdot \sin(60)$

$\boxed{h = \dfrac{\sqrt{3}}{2} x}$ (**)

$b_2 = b_1 + 2y$

$= 60 - 2x + 2\left(\tfrac{1}{2}x\right)$

$= 60 - 2x + x$

$\boxed{b_2 = 60 - x}$ (***)

Sub (*), (**), (***) in A

$A = \frac{1}{2}(b_1 + b_2)h$ (3 variables! we need 1)

$A(x) = \frac{1}{2}\left(60 - 2x + 60 - x\right)\left(\frac{\sqrt{3}}{2}x\right)$

$A(x) = \frac{\sqrt{3}}{4}x\left(120 - 3x\right)$

$A(x) = 30\sqrt{3}\,x - \frac{3\sqrt{3}}{4}x^2$

$A'(x) = 30\sqrt{3} - \frac{3\sqrt{3}}{2}x$

set to zero for c.v.

$\Rightarrow 30\sqrt{3} - \frac{3\sqrt{3}}{2}x = 0 \qquad \div 3\sqrt{3}$

$10 - \frac{1}{2}x = 0$

$\therefore x = 20$ cm

Test $x = 0, 20, 30$ in $A(x)$

$\Downarrow$

$A(0) = 0$

$A(20) = 519.6\ cm^2$

$A(30) = 389.7\ cm^2$

$\therefore$ The max area has dimensions of $\begin{cases} b_1 = 60 - 2(20) \\ \quad = 20\ cm \\ b_2 = 60 - (20) \\ \quad = 40\ cm \\ h = \frac{\sqrt{3}}{2}(20) = 10\sqrt{3}\ cm. \end{cases}$

b) Volume = (area of base)(height)

$= (519.6\ cm^2)(500\ cm)$

dimensions musts be same

$= 259800\ cm^3$

$= (0.2598\ m^3)$