Hwk Check 3.2

7. The fuel efficiency, E, in litres per 100 kilometres, for a car driven at speed v, in km/h, is $E(v) = \dfrac{1600v}{v^2 + 6400}$.

   a. If the speed limit is 100 km/h, determine the legal speed that will maximize the fuel efficiency.

   b. Repeat part a., using a speed limit of 50 km/h.

   c. Determine the speed intervals, within the legal speed limit of 0 km/h to 100 km/h, in which the fuel efficiency is increasing.

   d. Determine the speed intervals, within the legal speed limit of 0 km/h to 100 km/h, in which the fuel efficiency is decreasing.

derivative! $= 0$!

$$E'(v) = \frac{1600(v^2+6400) - 1600v(2v)}{(v^2+6400)^2}$$

$$= \frac{-1600v^2 + (1600)(6400)}{(v^2+6400)^2}$$

set $E'(v) = 0$

$$\Rightarrow \frac{-1600v^2 + (1600)(6400)}{(v^2+6400)^2} = 0$$

$$\Rightarrow -1600v^2 + (1600)(6400) = 0 \qquad \div (-1600)$$

$$\Rightarrow v^2 - 6400 = 0 \Rightarrow v = \pm 80$$

Test

$v = 0$

$v = 100$

$v = 80$

$E(0) = 0$

$E(100) = 9.8$

$E(80) = 10$

$\therefore$ 80 km/hr maximizes

b) $v \in [0, 50]$, $80 \notin [0, 50]$

$\therefore$ test $v = 0$ $\quad E(0) = 0$

$v = 50$ $\quad E(50) = 8.99$

$\therefore$ 50 is more efficient than 0 km/hr.

c) Fuel efficiency would be increasing if $E'(v) > 0$

$$\Rightarrow \frac{-1600v^2 + (1600)(6400)}{(v^2 + 6400)^2} > 0 \qquad \because (v^2 + 6400)^2 > 0 \quad \forall v \in [0, 100]$$

$$\Rightarrow -1600v^2 + 1600(6400) > 0 \qquad \div (-1600)$$

$$\Rightarrow v^2 - 6400 < 0$$

$$v^2 < 6400$$

$$|v| < 80 \Rightarrow -80 < v < 80$$

but $v \in [0, 100]$

$\therefore$ Efficiency is increasing for $0 \leq v < 80$

d) decreasing $80 < v < 100$.

8. The concentration $C(t)$, in milligrams per cubic centimetre, of a certain medicine in a patient's bloodstream is given by $C(t) = \frac{0.1t}{(t+3)^2}$, where t is the number of hours after the medicine is taken. Determine the maximum and minimum concentrations between the first and sixth hours after the medicine is taken.

$D_c = [1, 6]$

$$C'(t) = \frac{(0.1)(t+3)^2 - (0.1t)(2(t+3))}{(t+3)^4}$$

$$= \frac{0.1(t+3) - 0.2t}{(t+3)^3}$$

$$= \frac{-0.1t + 0.3}{(t+3)^3}$$

set to zero ($\Rightarrow$ numerator is zero)

$\Rightarrow -0.1t + 0.3 = 0$

$\Rightarrow t = 3.$

Test $t = 1$

$t = 3$

$t = 6$

$$C(1) = \frac{0.1(1)}{(1+3)^2} = \frac{1}{160}$$

$$C(3) = \frac{1}{120}$$

$$C(6) = \frac{1}{135}$$

$\therefore$ max concentration at 3 hours

min concentration at 1 hour.

9. Technicians working for the Ministry of Natural Resources found that the amount of a pollutant in a certain river can be represented by $P(t) = 2t + \frac{1}{(162t + 1)}$, $0 \leq t \leq 1$, where t is the time, in years, since a cleanup campaign started. At what time was the pollution at its lowest level?

$$P'(t) = 2 - \frac{162}{(162t+1)^2}$$

set to zero

$$\Rightarrow \quad 2 - \frac{162}{(162t+1)^2} = 0$$

$$\Rightarrow \quad (162t+1)^2 = 81$$

$$\Rightarrow \quad 162t + 1 = \pm 9$$

(negative 9 is ridiculous)

$$\Rightarrow \quad 162t + 1 = 9$$

$$\Rightarrow \quad t = \frac{8}{162} = 0.05$$

Test $t=0$, $t=0.05$, $t=1$ in $P(t)$ etc

10. A truck travelling at x km/h, where $30 \leq x \leq 120$, uses gasoline at the rate of $r(x)$ L/100 km, where $r(x) = \frac{1}{4}\left(\frac{4900}{x} + x\right)$. If fuel costs \$1.15/L, what speed will result in the lowest fuel cost for a trip of 200 km? What is the lowest total cost for the trip?

$$r'(x) = \frac{1}{4}\left(-\frac{4900}{x^2} + 1\right)$$

set to zero

$$\Rightarrow \frac{1}{4}\left(-\frac{4900}{x^2} + 1\right) = 0$$

$$\Rightarrow -\frac{4900}{x^2} + 1 = 0$$

$$\Rightarrow x^2 = 4900$$

$$\Rightarrow x = \pm 70$$ but -70 is crazy

$$\therefore x = 70$$

Test $x = 30, 70, 120$ in $r(x)$

$r(30) = 48.3$

$r(70) = 35$

$r(120) = 40.2$

at 70 km/hr we have lowest fuel consumption

$$(35 \text{ L}/100\text{ km})(200\text{ km}) = 70 \text{ L}$$

at \$1.15/L the cost is $(70)(1.15) = \$80.50$

Ben S. – 50,000 for being mean to my wife.

3.3 Optimization

The word "optimization" carries with it a number of meanings. In Mathematics, to **optimize** means to **find the "best" solution** to some real world problem. In terms of functions, the best solution is most often **a maximum or a minimum.**

Some argue that optimal method for learning about optimization is by simply jumping in and doing it. And so…

An Algorithmic Procedure for finding the Extrema of Functions

1. **Read** the problem carefully and determine what aspect of reality is being optimized.

2. **Write down all relevant functions and/or equations** which describe the problem. Pictures will help! (keep in mind "intervals of existence")
   2.1. **Note**: The function you wish to optimize may have two independent variables. You need to reduce to a single independent variable by using some "additional information".

3. **Differentiate the function** which is describing the problem **and find the critical values** (where the derivative is zero.)
   3.1. **Note**: The critical values will be at either a max or a min. We need to figure out which we have!

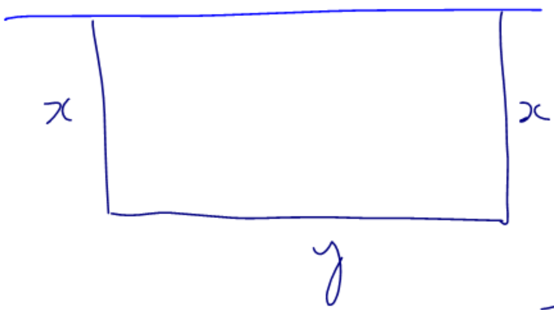
4. **Test the critical values** (and endpoints for a closed domain) to **determine the optimal value.**

5. **ANSWER THE QUESTION** (This may seem obvious, but make sure you answer the question that is asked!)

Example 3.3.1

From your text – Pg. 145 #3

A farmer has 600 m of fence and wants to enclose a rectangular field beside a river. Determine the dimensions of the fenced field in which the maximum area is enclosed. (Fencing is required on only three sides: those that aren't next to the river.)



$A = xy$

Problem w/ the fn

2 variables!!

so how do we take the derivative??

We need to "eliminate" one of the variables using the "other info"

We know

$600 = 2x + y$

$\boxed{y = 600 - 2x}$

(sub into area!)

$A = x(600 - 2x)$

$\Rightarrow \boxed{A = 600x - 2x^2}$

C.V. $\Rightarrow A' = 0$

$A' = 600 - 4x$

set to zero

$\Rightarrow 4x - 600 = 0$

$\Rightarrow x = 150$

$0 < x < 300$

(these end points would give an area of 0)

Test $x = 0, 150, 300$ in $A(x)$

these two are ridiculous

$A(0) = 0$

$A(150) = 45\,000 \text{ m}^2$

$A(300) = 0$

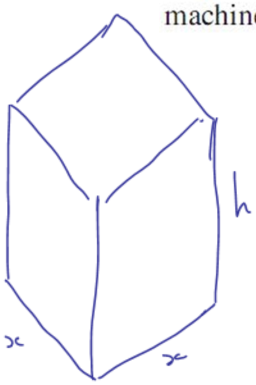
"$y = 600 - 2(150)$
$= 300$

$\therefore$ A field with dimensions 150 m by 300 m will maximize the area of the field.

Example 3.3.2

From your text – Pg. 146 #9

The volume of a square-based rectangular cardboard box needs to be 1000 cm^3. Determine the dimensions that require the minimum amount of material to manufacture all six faces. Assume that there will be no waste material. The machinery available cannot fabricate material smaller than 2 cm in length.



Note: we minimize surface area (there 6 surfaces!)

top + bottom

4 sides

$$SA = 2(x^2) + 4(xh)$$

$$SA = 2x^2 + 4xh$$

$V = 1000 \text{ cm}^3$

$\therefore x^2 h = 1000$

$h = \frac{1000}{x^2}$ (*)

(sub into SA)

$$\Rightarrow SA = 2x^2 + 4x\left(\frac{1000}{x^2}\right)$$

$$\Rightarrow SA = 2x^2 + \frac{4000}{x}$$

$$SA' = 4x - \frac{4000}{x^2}$$

C.N. $\Rightarrow SA' = 0$

$$\Rightarrow 4x - \frac{4000}{x^2} = 0$$

$$4x = \frac{4000}{x^2}$$

$$4x^3 = 4000 \Rightarrow x = 10$$

$\times x^2$

$4x^3 - 4000 = 0$

$4x^3 = 4000$

$x^3 = 1000$

$x = 10$

$2 \leq x \leq \sqrt{500}$

$h \geq 2$

$\frac{1000}{x^2} \geq 2$

$\Rightarrow 2x^2 \leq 1000$

$x^2 \leq 500$

$x \leq \sqrt{500}$

(next pg)

Class/Homework for Section 3.3

Pg. 145 #4 - 8

wde hurt my feelings

Test $x = 2, 10, \sqrt{500}$

$SA(2) = 2008$

$SA(10) = 600$

$SA(\sqrt{500}) = 1000 + \frac{4000}{\sqrt{500}} > 600$

$\therefore$ The dimensions of the box which minimize SA are $w = 10$, $l = 10$, $h = \frac{1000}{(10)^2} = 10$ (by (*))