3.3b More Optimization Examples

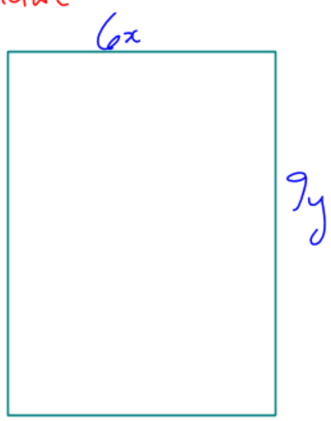
We've talked about the fact that real world problems can very often be described mathematically. When the mathematical description includes functions, then Calculus becomes a very powerful tool. **We need to keep in mind, though, that the functions we use to describe the real world will necessarily have "restricted" domains**. The restrictions to the domains arise from **constraints imposed by real world conditions**. Understanding and incorporating these constraints into the mathematical descriptions of our problems is an art worth learning, and **must be used if we hope to employ the EVT**!

Example 3.3.3

From your text: Pg. 152 #5

A rectangular piece of land is to be fenced using two kinds of fencing. Two opposite sides will be fenced using standard fencing that costs \$6/m, while the other two sides will require heavy-duty fencing that costs \$9/m. What are the dimensions of the rectangular lot of greatest area that can be fenced for a cost of \$9000?

Picture



$\text{Cost} = \$6(2x) + \$9(2y)$

$\Rightarrow 9000 = 12x + 18y$

$\Rightarrow y = \dfrac{1500 - 2x}{3} \quad (*)$

(after cancelling a common "6")

$0 \le x \le 750$

Note: for $x = 0$ or $x = 750$ we get a minimum area

$A = xy$

$\Rightarrow A(x) = x\left(\dfrac{1500 - 2x}{3}\right)$ by $(*)$

$\Rightarrow A(x) = 500x - \frac{2}{3}x^2$ (quadratic $\Rightarrow$ max is at c.v.!)

$\Rightarrow A'(x) = 500 - \frac{4}{3}x$

set to zero for c.v.

$\Rightarrow 500 - \frac{4}{3}x = 0$

$\Rightarrow x = 375$ (max area is at critical value)

$\therefore$ Area is maximized by a fence w/ dimensions:

found using $(*)$

375 m of \$6 fence by 250 m of \$9 fence

Example 3.3.4

From your text: Pg. 152 #7

A bus service carries 10 000 people daily between Ajax and Union Station, and the company has space to serve up to 15 000 people per day. The cost to ride the bus is \$20. Market research shows that if the fare increases by \$0.50, 200 fewer people will ride the bus. What fare should be charged to get the maximum revenue, given that the bus company must have at least \$130 000 in fares a day to cover operating costs?

"Picture"

Bus fare	Riders
\$20.00	15 000
\$20.50	14 800
\$21.00	14 600

etc

let x be the number of \$0.50 increases in fare

$(20 + 0.50x)$ ⋮ $(15000 - 200x)$

Revenue = (cost of a ticket)(# of riders)

$\Rightarrow R(x) = (20 + 0.5x)(15000 - 200x)$ ← quadratic "opening down" $\Rightarrow$ C.V. is max

(Note: we need $R(x) \geq \$130\,000$ - will have to check)

← product rule

$\Rightarrow R'(x) = 0.5(15000 - 200x) + (20 + 0.5x)(-200)$

$\Rightarrow R'(x) = -200x + 3500$

set to zero for C.V.

$\Rightarrow -200x + 3500 = 0$

$\Rightarrow x = 17.5$ (this C.V. gives max)

(Here we are faced with an issue - x is the number of \$0.50 fare increases. With $x = 17.5$ do we increase the fare by $(17.5)(0.50) = \$8.75$, or do we check $x = 17$ & $x = 18$, keeping the # of increases in fare a whole number?)

my personal feeling is that it's financially reasonable to go for the actual max, and so use $x = 17.5$. Some "real world" problems may not allow for decimals (eg x represents # of people) in which case you check integer values nearest to your 'decimal' max.

Q. Does $x = 17.5$ give a revenue of at least \$130 000?

$R(17.5) = \$330\,625$

$\therefore$ A fare of $\$20 + \$0.5(17.5) = \boxed{\$28.75}$ should be charged to maximize revenue.

Some info missing - the smallest a dimension can be.

Assume "r" or "h" ≥ 1 → "reasonable" smallest radius.

NOTE: The 'cost' of tin is unknown
may "wall tin" is \$10/cm²
⇒ "end tin" is \$20/cm² - whatever

Example 3.3.5

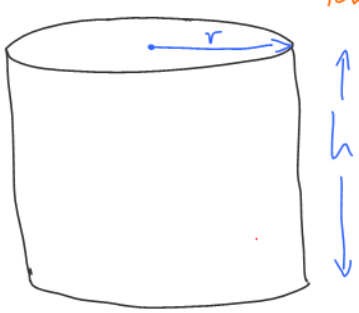
From your text: Pg. 153 #10

The cost of producing an ordinary cylindrical tin can is determined by the materials used for the wall and the end pieces. If the end pieces are twice as expensive per square centimetre as the wall, find the dimensions (to the nearest millimetre) to make a 1000 cm³ can at minimal cost.

Let 'c' be the cost of "wall" tin per mm². NOT a variable!!
⇒ 2c is cost of "end tin".

Picture

Volume "links" the variables



Cost = cost of top & bottom + cost of side

$$\Rightarrow C = (2c)(2\pi r^2) + (c)(2\pi r h)$$

The variables are r & h
"c" is just an unknown number.

$$V = \pi r^2 h$$

$$\Rightarrow 1000 = \pi r^2 h$$

$$\Rightarrow h = \frac{1000}{\pi r^2} \quad (*)$$

sub into C

$$1 \leq r \leq \sqrt{\frac{1000}{\pi}} = 6.83$$

(since $h \geq 1$ too)

$$\Rightarrow C(r) = 4c\pi r^2 + 2c\pi r\left(\frac{1000}{\pi r^2}\right)$$

$$\Rightarrow C(r) = 4c\pi r^2 + \frac{2000c}{r}$$

(NOT a quadratic ⇒ C.V. may not give min)

$$\Rightarrow C'(r) = 8c\pi r - \frac{2000c}{r^2}$$

set to zero for C.V.

remember, "c" is just a number!

$$\Rightarrow 8c\pi r - \frac{2000c}{r^2} = 0 \quad (\div 8c)$$

$$\Rightarrow \pi r - \frac{250}{r^2} = 0$$

(we didn't actually need to know the "base cost" of the tin "c")

Class/Homework for Section 3.3b

Pg. 151 – 154 #3 – 7, 9 – 11, 14, 15

next pg.

$$\Rightarrow \pi r^3 = 250$$

$$\Rightarrow r = \left(\frac{250}{\pi}\right)^{\frac{1}{3}} = 4.30 \text{ cm.}$$

Test $r = 1, 4.30, 6.83$

again "c" might be something like \$10/cm^2

$$C(1) = (4\pi + 2000)c = 2012.6c$$

$$C(4.30) = 697.5c$$

$$C(6.83) = 879.0c$$

$\therefore$ The dimensions of the can which minimize cost are

$$r = 4.3 \text{ cm} \quad \text{and} \quad h = \frac{1000}{\pi(4.3)^2} = 17.2 \text{ cm.} \quad (\text{by } (*))$$

I hope all the above makes sense.

Happy days one and all.