CALCULUS

Chapter 4 –Curve Sketching

(*Material adapted from Chapter 4 of your text*)

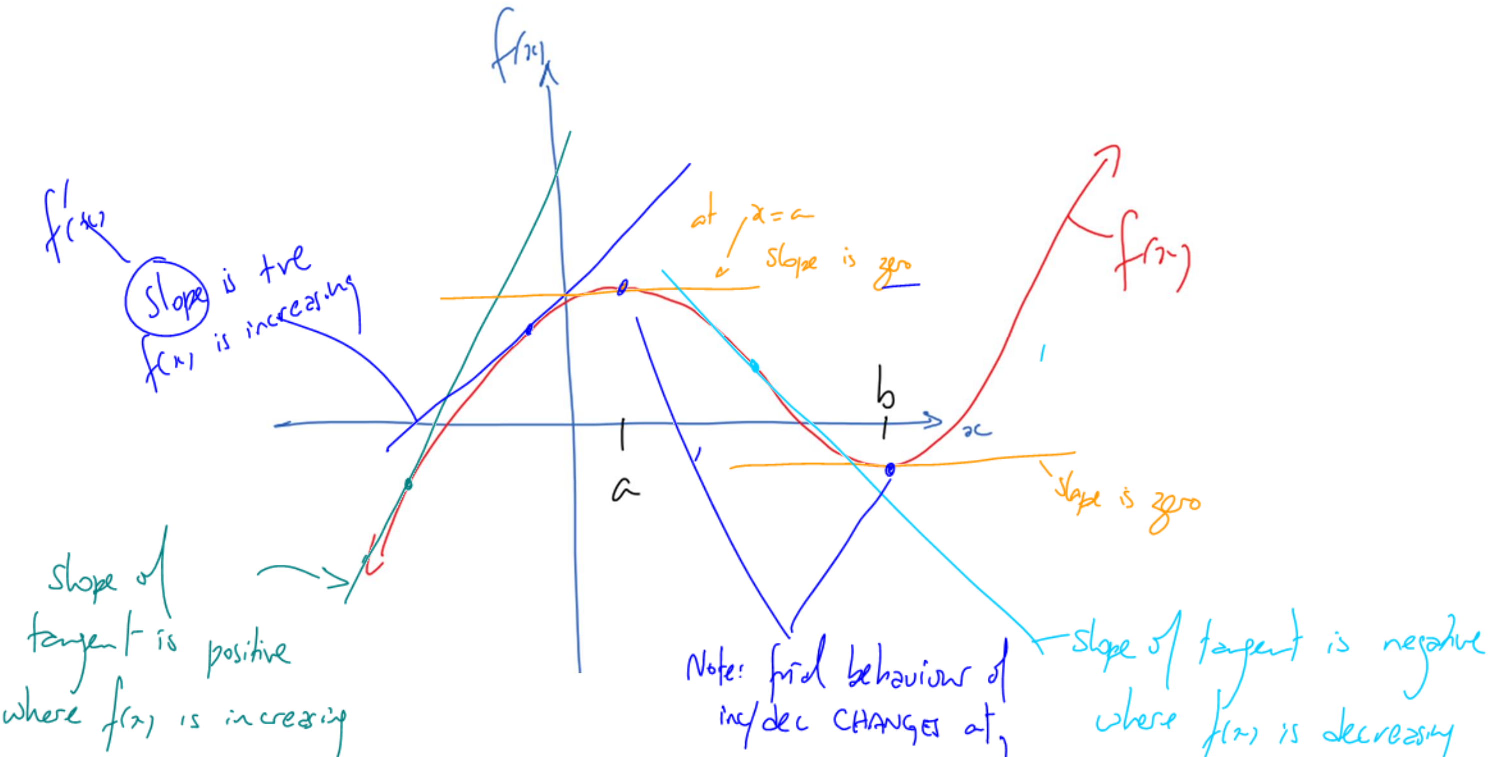
A∞Ω

MATH@TD

4.1 Intervals of Functional Increase and Decrease

In this chapter we will concern ourselves a little more with **Functional Behaviour.**

Consider the picture:



Clearly, $f(x)$ is increasing on $x \in (-\infty, a) \cup (b, \infty)$

and decreasing on $x \in (a, b)$

Note: At $x = a$, $x = b$ the fn $f(x)$ is neither increasing nor decreasing. At $x = a$, $x = b$ we have "turning points" or critical values.

At critical values, find behaviour of inc/dec may change

Definition 4.1.1

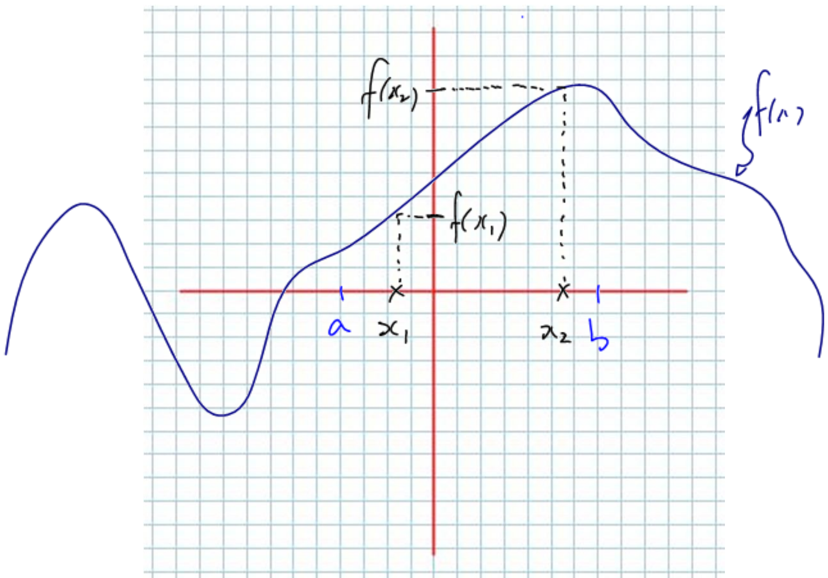
1) A function $f(x)$ is said to be **increasing** on the **open interval** (a,b) if

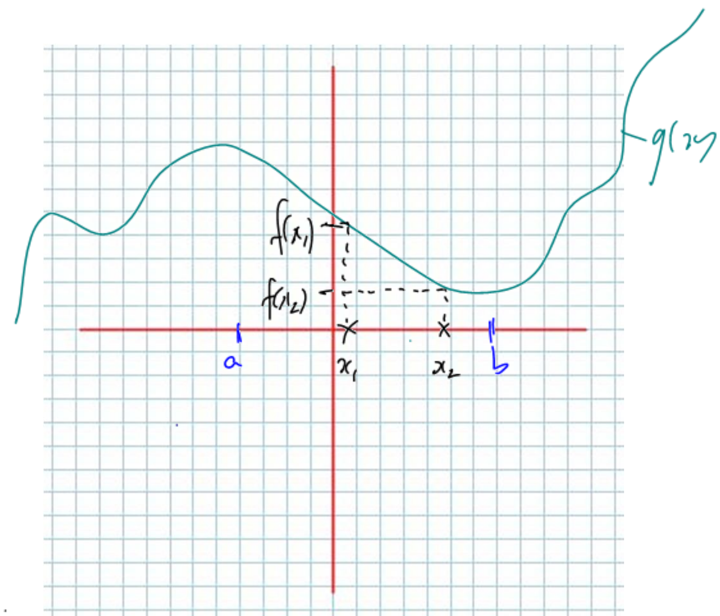
$$f(x_2) > f(x_1) \quad \text{whenever} \quad x_1 < x_2 \quad \text{for} \quad x_1, x_2 \in (a,b)$$

2) A function $g(x)$ is said to be **decreasing** on the **open interval** (a,b) if

$$g(x_2) < g(x_1) \quad \text{whenever} \quad x_1 < x_2 \quad \text{for} \quad x_1, x_2 \in (a,b)$$

Pictures



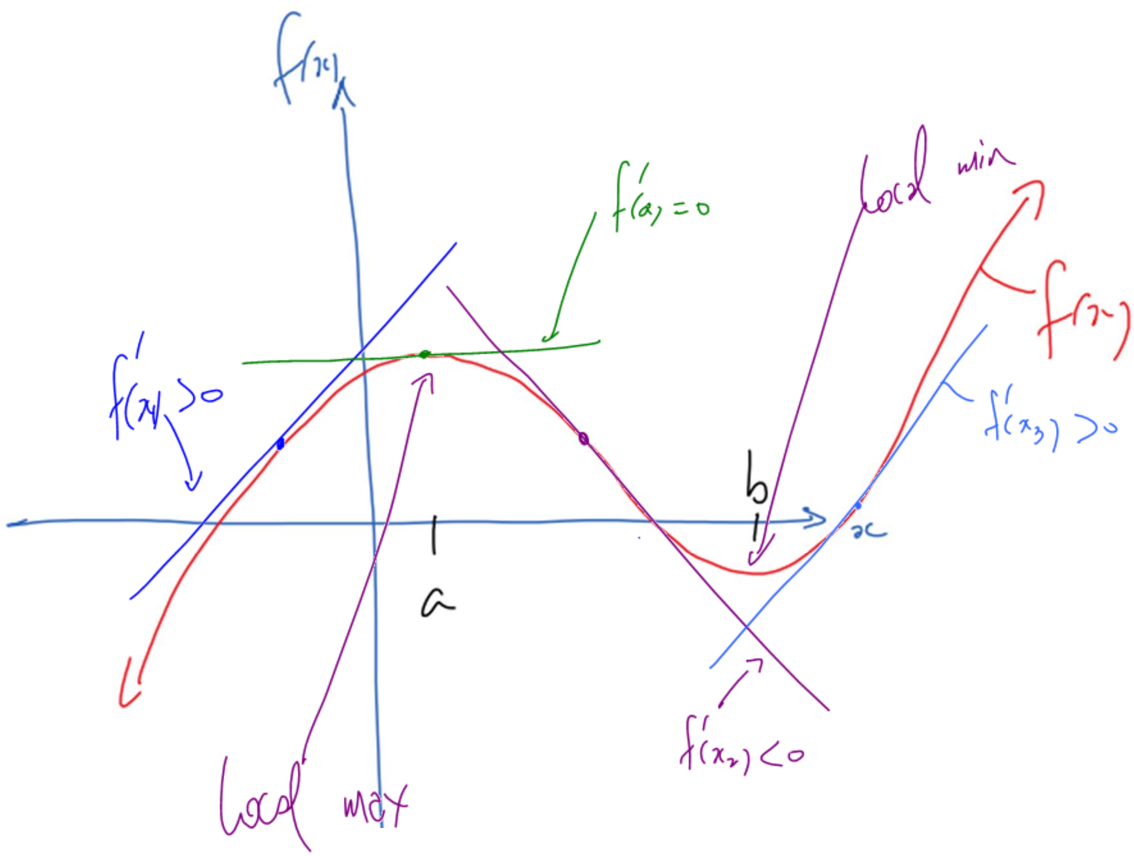


While Definition 4.1.1 is true, it's not very "fun" to work with. Perhaps there is something better!

The First Derivative Test (for local max/min)

Given some differentiable function, $f(x)$, we can **use its first deriva**tive **to determine** where the function is **increasing and decreasing**. Furthermore, we can **use that information to test whether a critical value is the location of a local maximum or a local minimum** (more on that later).

Picture



over a chunk of the domain

Definition 4.1.2 (Calculus point of view)

Given a differentiable function, $f(x)$, whenever $f'(x) > 0$, $f(x)$ is increasing

Whenever $f'(x) < 0$, $f(x)$ is decreasing

Example 4.1.1

Determine the intervals of increase and decrease for the polynomial function

$f(x) = x^5 - 5x^4 + 100$

$f'(x) = 5x^4 - 20x^3$ set to zero for C.V.

$\Rightarrow 5x^4 - 20x^3 = 0 \quad \div 5$

$\Rightarrow x^4 - 4x^3 = 0$

$\Rightarrow x^3(x-4) = 0$

$\therefore x = 0$ or $x = 4$

Note: we want the critical values because at these values the final behaviour of inc/dec changes

Q. How do we find intervals of inc/dec?

Interval Chart!

Intervals	$(-\infty, 0)$	$(0, 4)$	$(4, \infty)$
Test Value	-1	1	5
Sign on $f'(x) = 5x^4 - 20x^3$	+ve	−ve.	+ve
inc/dec for $f(x)$	inc ↗	dec ↘	inc ↗

NOTE $x = 0$ is a local max

$x = 4$ is a local min!

$\therefore f(x)$ is increasing on $x \in (-\infty, 0) \cup (4, \infty)$

" " dec " $x \in (0, 4)$

Example 4.1.2

Determine the intervals of increase and decrease for the function $g(x) = x + \frac{1}{x}$.

$g'(x) = 1 - \frac{1}{x^2}$ set to zero for C.V.

$\hookrightarrow x = 0$ is NOT in the domain

$\Rightarrow 1 - \frac{1}{x^2} = 0$

$1 = \frac{1}{x^2}$

$\Rightarrow x^2 = 1 \Rightarrow |x| = 1$

$\Rightarrow x = \pm 1$

$x = \pm 1$

of the domain

asymptote

INTERVALS	$(-\infty, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \infty)$
TEST VALS	-2	$-\frac{1}{2}$	$\frac{1}{2}$	2
SIGN on $g'(x) = 1 - \frac{1}{x^2}$	+ve	−ve	−ve	+ve
inc/dec for $g(x)$	inc	dec	dec	inc

$x = -1$ is a local max

$x = 1$ is a local min

$\therefore g(x)$ is inc on $x \in (-\infty, -1) \cup (+1, \infty)$

" " dec " $x \in (-1, 0) \cup (0, 1)$

Class/Homework for Section 4.1

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