Hannah left her lights on.

4.2 Critical Values and Local Extrema

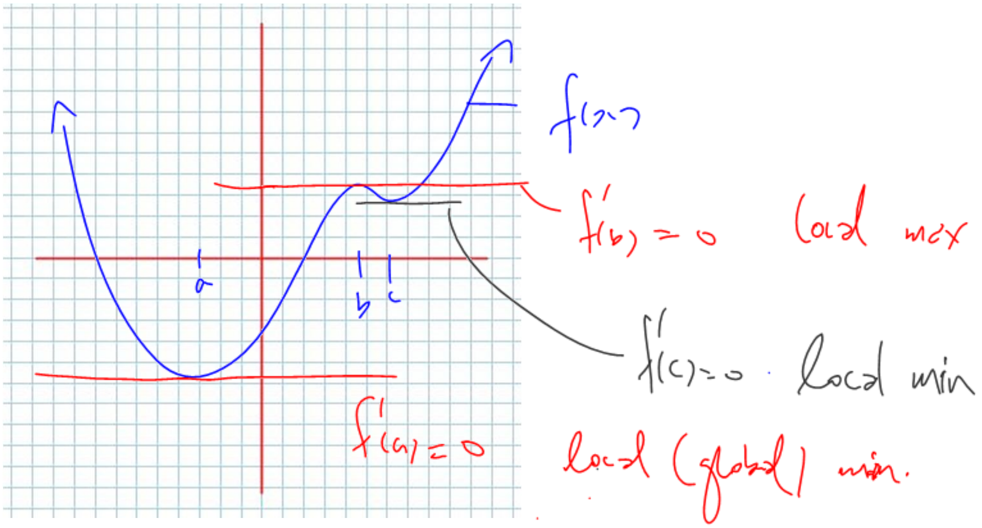
(Much of this is review)

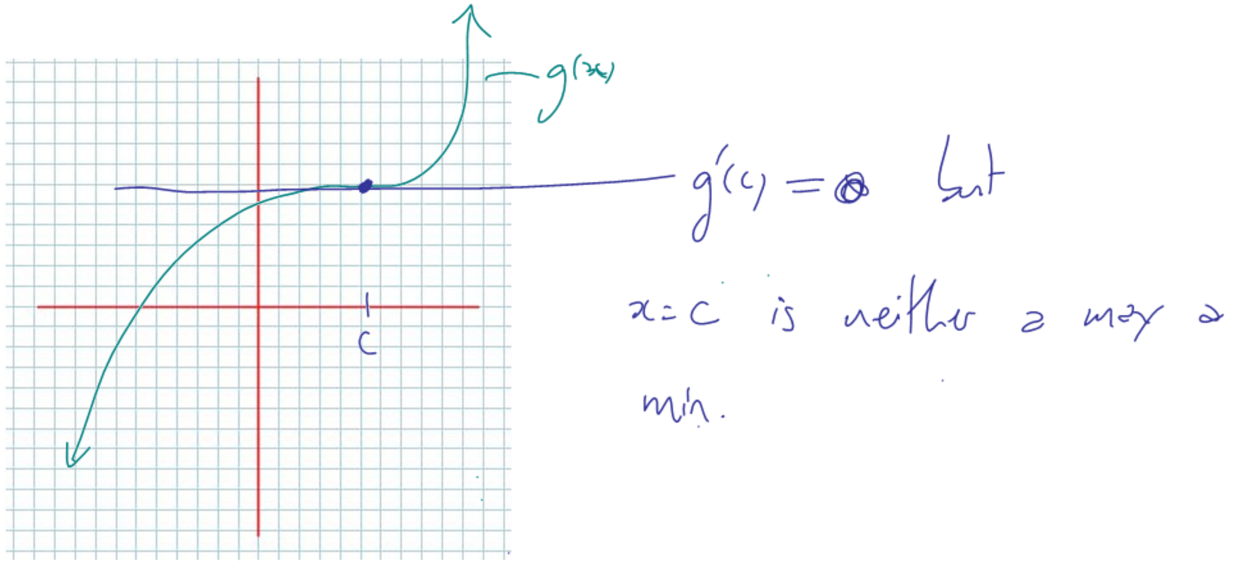
Recall: an **extremum** (an extreme value) is either a **maximum** or a **minimum**.

Definition 4.2.1

Given a differentiable function, $f(x)$, at any domain values $x = c$, where $f'(c) = 0$, then $f(x)$ **MAY** have **local extrema** at $x = c$.

Pictures





x = c is neither a max or min.

test for local max/min.

The First Derivative Test *(more formally than in 4.1)*

Given a differentiable function, $f(x)$, where $f'(c)=0$, and if:

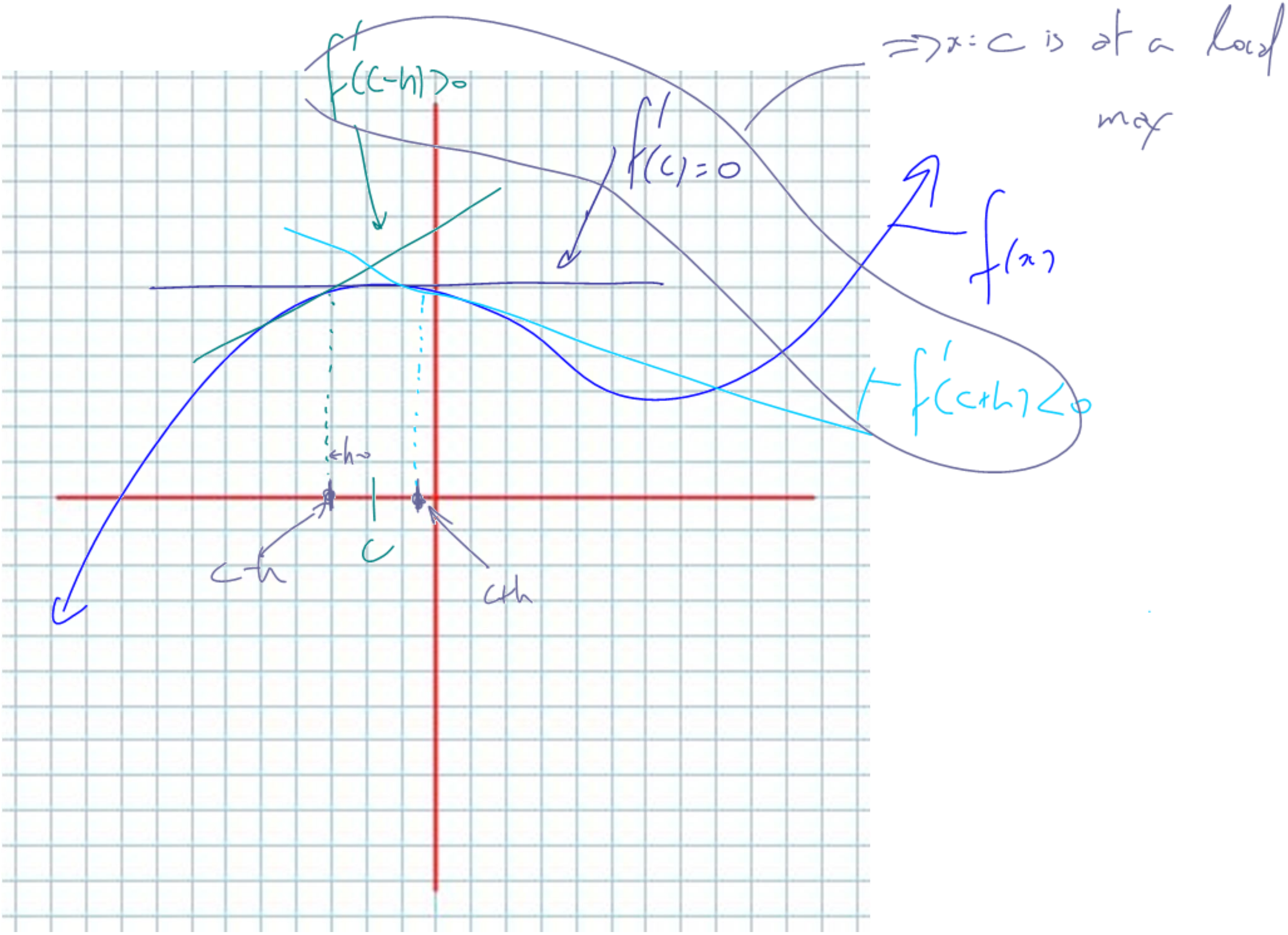
1) $f'(c-h)>0$ **AND** $f'(c+h)<0$ (where h is some ***small*** positive number)

   then $x=c$ is where $f(x)$ has a local maximum.

2) $f'(c-h)<0$ **AND** $f(c+h)>0$ (where h is some ***small*** positive number)

   Then $x=c$ is where $f(x)$ has a local minimum.

Picture



Definition 4.2.2

Given a differentiable function, $f(x)$, we say $x=c$ is a critical value of $f(x)$ if either:

$f'(c)=0$ OR $f'(c)$ d.n.e

Example 4.2.1

Determine the critical values of $f(x)=\frac{x^2-4}{x-3}$, and determine if $f(x)$ has any local extrema.

$$f'(x)=\frac{(2x)(x-3)-(x^2-4)}{(x-3)^2}$$

$$=\frac{x^2-6x+4}{(x-3)^2}$$

set to 0 for c.v.
or $f'(x)$ dne

$\Longrightarrow x^2-6x+4=0$ d.n.f.

$\Rightarrow x=3$ is c.v.
(V.A.)

Q.F. $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\Downarrow$

$x=3-\sqrt{5}$, $x=3+\sqrt{5}$

$\therefore$ The C.V.'s are $x=3-\sqrt{5}$, $x=3$, $x=3+\sqrt{5}$

Class/Homework for Section 4.2

Pg. 178 – 180 #1 – 5, 7cdef, 9, 10, 12 – 15

↓ Test for extrema.

INTERVAL CHART

V.A.

intervals	$(-\infty, 3-\sqrt{5})$	$(3-\sqrt{5}, 3)$	$(3, 3+\sqrt{5})$	$(3+\sqrt{5}, \infty)$
T.V.	0	1	4	6
Sign on $f'(x) = \frac{x^2-6x+4}{(x-3)^2}$	+ve	−ve	−ve	+ve
inc/dec	inc ↗	dec ↘	dec ↘	inc ↗

$\therefore$ $x = 3-\sqrt{5}$ is a local max

and $x = 3+\sqrt{5}$ is a local min.

$x = 3$ is a V.A.