4.1 Critical Values and Local Extrema

These problems are taken from the text, Pg. 178 – 180.

2. a. For the function $y = x^3 - 6x^2$, explain how you would find the critical points.
   b. Determine the critical points for $y = x^3 - 6x^2$, and then sketch the graph.

3. Find the critical points for each function. Use the first derivative test to determine whether the critical point is a local maximum, local minimum, or neither.

   a. $y = x^4 - 8x^2$ b. $f(x) = \dfrac{2x}{x^2 + 9}$ c. $y = x^3 + 3x^2 + 1$

7. Determine the critical points for each of the following functions, and determine whether the function has a local maximum value, a local minimum value, or neither at the critical points. Sketch the graph of each function.

   a. $f(x) = -2x^2 + 8x + 13$
   b. $f(x) = \dfrac{1}{3}x^3 - 9x + 2$
   c. $f(x) = 2x^3 + 9x^2 + 12x$
   d. $f(x) = -3x^3 - 5x$
   e. $f(x) = \sqrt{x^2 - 2x + 2}$
   f. $f(x) = 3x^4 - 4x^3$

9. Sketch a graph of a function f that is differentiable on the interval $-3 \leq x \leq 4$ and that satisfies the following conditions:

   - The function f is decreasing on $-1 < x < 3$ and increasing elsewhere on $-3 \leq x \leq 4$.
   - The largest value of f is 6, and the smallest value is 0.
   - The graph of f has local extrema at $(-1, 6)$ and $(3, 1)$.

10. Determine values of a, b, and c such that the graph of $y = ax^2 + bx + c$ has a relative maximum at $(3, 12)$ and crosses the y-axis at $(0, 1)$.

12. For $f(x) = x^3 - kx$, where $k \in \mathbf{R}$, find the values of k such that f has

a. no critical numbers b. one critical number c. two critical numbers

13. Find values of a, b, c, and d such that $g(x) = ax^3 + bx^2 + cx + d$ has a local maximum at (2, 4) and a local minimum at (0, 0).

15. Consider the function $f(x) = 3x^4 + ax^3 + bx^2 + cx + d$.

a. Find constants a, b, c, and d such that the graph of f will have horizontal tangents at $(-2, -73)$ and $(0, -9)$.

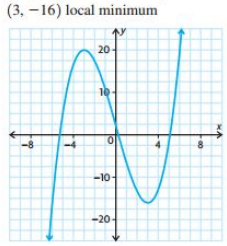
b. There is a third point that has a horizontal tangent. Find this point.

c. For all three points, determine whether each corresponds to a local maximum, a local minimum, or neither.

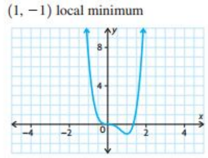
Answers to Selected Problems

3. **a.** local minima: $(-2, -16)$, $(2, -16)$, local maximum: $(0, 0)$
b. local minimum: $(-3, -0.3)$, local maximum: $(3, 0.3)$
c. local minimum: $(-2, 5)$, local maximum: $(0, 1)$

7. **b.** $(-3, 20)$ local maximum, $(3, -16)$ local minimum



f. $(0, 0)$ neither maximum nor minimum, $(1, -1)$ local minimum



10. $a = -\frac{11}{9}$, $b = \frac{22}{3}$, $c = 1$

15. **a.** $a = -4$, $b = -36$, $c = 0$
b. $(3, -198)$
c. local minimum: $(-2, -73)$ and $(3, -198)$, local maximum: $(0, -9)$