4.4 Concavity and Points of Inflection

(*These problems are taken from the Nelson Text: Pg. 205 – 206*)

2. Determine the critical points for each function, and use the second derivative test to decide if the point is a local maximum, a local minimum, or neither.
   a. $y = x^3 - 6x^2 - 15x + 10$
   b. $y = \dfrac{25}{x^2 + 48}$
   c. $s = t + t^{-1}$
   d. $y = (x - 3)^3 + 8$

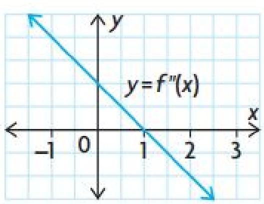
3. Determine the points of inflection for each function in question 2. Then conduct a test to determine the change of sign in the second derivative.

4. Determine the value of the second derivative at the value indicated. State whether the curve lies above or below the tangent at this point.
   a. $f(x) = 2x^3 - 10x + 3$ at $x = 2$
   b. $g(x) = x^2 - \dfrac{1}{x}$ at $x = -1$
   c. $p(w) = \dfrac{w}{\sqrt{w^2 + 1}}$ at $w = 3$
   d. $s(t) = \dfrac{2t}{t - 4}$ at $t = -2$

Note: The tangent lying above the curve would indicate a max, while a min would have the tangent below the curve.

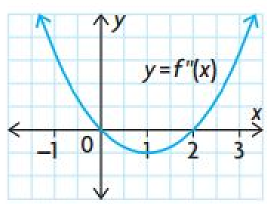
5. Each of the following graphs represents the second derivative, $f''(x)$, of a function $f(x)$:

a.



$f''(x)$ is a linear function.

b.



$f''(x)$ is a quadratic function.

For each of the graphs above, answer the following questions:

i. On which intervals is the graph of $f(x)$ concave up? On which intervals is the graph concave down?

ii. List the x-coordinates of all the points of inflection.

iii. Make a rough sketch of a possible graph of $f(x)$, assuming that $f(0) = 2$.

(over)

9. Sketch the graph of a function with the following properties:
   - $f'(x) > 0$ when $x < 2$ and when $2 < x < 5$
   - $f'(x) < 0$ when $x > 5$
   - $f'(2) = 0$ and $f'(5) = 0$
   - $f''(x) < 0$ when $x < 2$ and when $4 < x < 7$
   - $f''(x) > 0$ when $2 < x < 4$ and when $x > 7$
   - $f(0) = -4$

10. Find constants a, b, and c such that the function $f(x) = ax^3 + bx^2 + c$ will have a local extremum at $(2, 11)$ and a point of inflection at $(1, 5)$. Sketch the graph of $y = f(x)$.

11. Find the value of the constant b such that the function $f(x) = \sqrt{x+1} + \frac{b}{x}$ has a point of inflection at $x = 3$.

Answers to Selected Problems

2. **a.** local minimum: $(5, -105)$, local maximum: $(-1, 20)$

b. local maximum: $\left(0, \frac{25}{48}\right)$

c. local maximum: $(-1, -2)$, local minimum: $(1, 2)$

d. $(3, 8)$ is neither a local maximum or minimum.

4. **a.** 24; above

b. 4; above

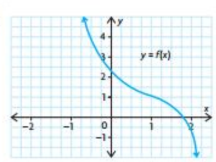
c. $-\frac{9}{100\sqrt{10}}$; below

d. $-\frac{2}{27}$; below

5. **a.** **i.** concave up on $x < 1$, concave down on $x > 1$

ii. $x = 1$

iii.



10. $a = -3, b = 9, c = -1$