CALCULUS

Chapter 5 –Trigonometric, Exponential and Logarithmic Functions

(*Material adapted from Chapter 5 of your text*)

A∞Ω

MATH@TD

5.1 The Derivative of $f(x)=e^x$

e is a **transcendental number**, as is the number π. *e* is named after **Leonard Euler**, who is one of the most brilliant humans to have ever walked the face of the earth.

$$e = 2.71828182845904523536028747135266249775724709369995...$$

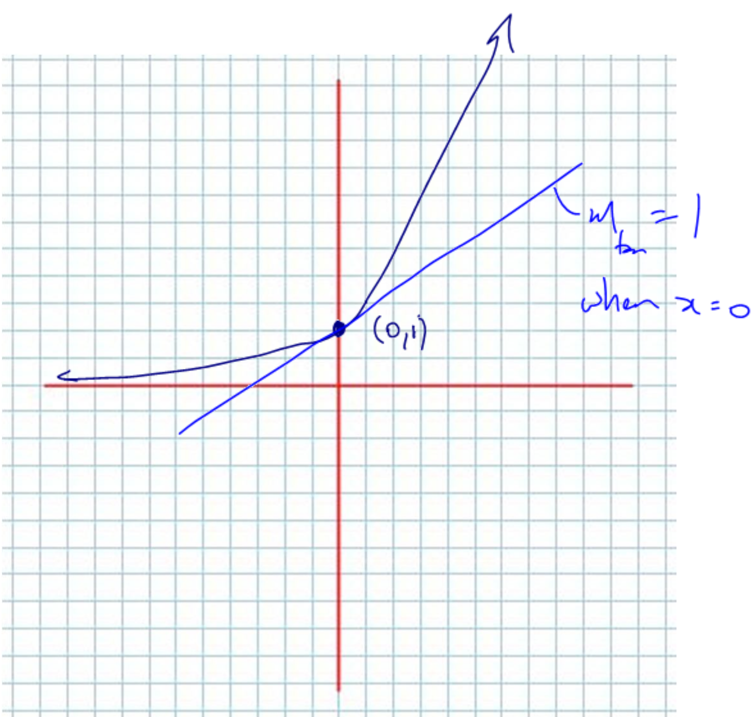
Definition 5.1.1

$$e=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n \quad \text{(obviously....)}$$

Some properties of the function $f(x)=e^x$

It is an exponential function, with base *e*.

Picture



Note $e>1$

$\lim_{x\to-\infty}(e^x) = 0$

$\Rightarrow y=0$ is a H.A

The Derivative

Given $f(x)=e^x$, then

$$\frac{df}{dx}(x)=f'(x)=\lim_{h\to 0}\left(\frac{e^{x+h}-e^x}{h}\right)=e^x\cdot\lim_{h\to 0}\left(\frac{e^h-1}{h}\right)$$

$$=e^x$$

Consider the **composite function** $f(x)=e^{g(x)}$. The by the ***Chain Rule***

"outer" is "e^x"

inner is $g(x)$

$$f'(x)=e^{g(x)}\cdot g'(x)=g'(x)\cdot e^{g(x)}$$

(itself) times (the derivative of power)

Example 5.1.1

Differentiate:

a) $y=e^{3x^2}$

$$y'=e^{3x^2}(6x)$$

$$=6x\,e^{3x^2}$$

b) $f(x)=e^{\sqrt[3]{x^2-4}}$

$$f'(x)=e^{\sqrt[3]{x^2-4}}\left(\frac{1}{3}(x^2-4)^{-2/3}\cdot 2x\right)$$

$$=\frac{2x\,e^{\sqrt[3]{x^2-4}}}{3(x^2-4)^{2/3}}$$

Example 5.1.2

From your text: Pg. 232 #3

Differentiate:

c) $f(x) = \dfrac{e^{-x^3}}{x}$ quotient + chain

$$f'(x) = \frac{(e^{-x^3})(-3x^2)(x) - e^{-x^3}(1)}{x^2}$$

$$= \frac{e^{-x^3}\left(-3x^3 - 1\right)}{x^2}$$

b) $y = x \cdot e^{3x}$ product + chain

$$y' = (1)e^{3x} + (x)\left(e^{3x} \cdot 3\right)$$

$$= e^{3x}\left(1 + 3x\right)$$

d) $g(x) = \sqrt{x} \cdot e^{x}$ product

$$g'(x) = \frac{1}{2\sqrt{x}} e^{x} + \sqrt{x} \cdot e^{x}$$

$$= e^{x}\left(\frac{1}{2\sqrt{x}} + \frac{\sqrt{x}}{1}\right)$$

$$= e^{x}\left(\frac{1 + 2x}{2\sqrt{x}}\right)$$

form $y = mx + b$ ← from the point A

from derivative

Example 5.1.3

From your text: Pg. 233 #7

Determine the equation of the tangent to the curve $y = x \cdot e^{-x}$ at the point $A\left(1, e^{-1}\right)$.

$$y' = (1)e^{-x} + (x)(-e^{-x})$$
$$= e^{-x}(1 - x)$$

$$m_{tan} = y'\Big|_{x=1} = e^{-1}(1-1)$$
$$= 0$$

$\therefore$ eqn is $y = 0x + b$

$\Rightarrow y = b$ use $A(1, e^{-1})$

$e^{-1} = b \Rightarrow$

$$\boxed{y = e^{-1}}$$

a horizontal line

$\Rightarrow$ Local max/min at $A(1, e^{-1})$

Class/Homework for Section 5.1

Pg. 232 – 233 #2 – 4, 6, 9, 11 – 13