5.2 The Derivative of the General Exponential

In section 5.1 we learned that for the (so called) Natural Exponential Function $f(x) = e^x$ the derivative is given by: $f'(x) = e^x$

base "e"

We now turn our attention to the General Exponential Function $f(x) = b^x$, $b > 0$ and we ask "**what is its derivative?**"

Before answering that question it will be helpful to review a little bit about Exponentials and their inverses: Logarithms.

Given an exponential equation, $y = 2^x$ we can invert using a logarithm and isolate for x:

$\Rightarrow \log_2(y) = x$

So for $y = e^x$

$\log_e(y) = x$

Finally, recall that $\log_a(a) = 1$

And so $\log_e(e) = \ln(e) = 1$

Example 5.2.1

Given $g(x) = 2^x$ determine $g'(x)$.

If only it was base e

Change to base "e"

Take "ln" of both sides

$\ln(g(x)) = \ln(2^x) \Rightarrow \ln(g(x)) = x \cdot \ln(2)$

$\Rightarrow g(x) = e^{x \cdot \ln(2)}$

$\log_e(g) = \text{stuff}$

$g = e^{\text{stuff}}$

$\boxed{e^{\ln(a)} = a}$

$g'(x) = e^{x \cdot \ln(2)} \cdot \ln(2)$

$= e^{\ln(2^x)} \cdot \ln(2)$

$= 2^x \cdot \ln(2)$

The derivative of an exponential is ITSELF, Times the ln of the base

In general, given an exponential function $f(x) = b^x,\ b > 0$, then

$$f'(x) = b^x \cdot \ln(b)$$

— accounts for the non-base e

Consider the composite function $f(x) = b^{g(x)}$.

$$f'(x) = b^{g(x)} \cdot \ln(b) \cdot g'(x)$$

Chain Rule

Example 5.2.2

From your text: Pg 240 #1. Differentiate:

a) $y = 2^{3x}$

$$y' = 2^{3x} \cdot \ln(2) \cdot 3$$
$$= 2^{3x} \cdot 3\ln(2)$$

d) $w = 10^{\left(5-6n+n^2\right)}$

$$\frac{dw}{dn} = 10^{(5-6n+n^2)} \cdot \ln(10) \cdot (2n-6)$$

Example 5.2.3

From your text: Pg. 240 #2b) Differentiate

$y = x \cdot 3^{x^2}$

$$y' = (1) \cdot 3^{x^2} + x \cdot \left(3^{x^2} \cdot \ln(3) \cdot (2x)\right)$$
$$= 3^{x^2}\left(1 + 2x^2 \cdot \ln(3)\right)$$

Class/Homework for Section 5.2

Pg. 240 # 1 – 7