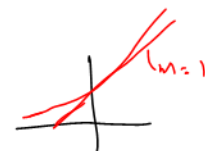


5.2 The Derivative of the General Exponential



In section 5.1 we learned that for the (so called) Natural Exponential Function $f(x) = e^x$ the derivative is given by: $f'(x) = e^x$ base e

We now turn our attention to the General Exponential Function $f(x) = b^x$, $b > 0$ and we ask “what is its derivative?”

Before answering that question it will be helpful to review a little bit about Exponentials and their inverses: Logarithms.

Given an exponential equation, $y = 2^x$ we can invert using a logarithm and isolate for x :

$$\Rightarrow \log_2(y) = x$$

So for $y = e^x$

$$\log_e(y) = x$$

Finally, recall that $\log_a(a) = 1$

$$\text{And so } \log_e(e) = \ln(e) = 1$$

Example 5.2.1

Given $g(x) = 2^x$ determine $g'(x)$.

If only it was base e

Change to base e

Take “ln” of both sides

$$\ln(g(x)) = \ln(2^x) \Rightarrow \ln(g(x)) = x \cdot \ln(2)$$

$$\Rightarrow g(x) = e^{x \cdot \ln(2)}$$

$$\log_e(g) = \text{stuff}$$

$$g = e^{\text{stuff}}$$

$$\begin{array}{|c|} \hline \ln(a) \\ \hline e \\ \hline = a \\ \hline \end{array}$$

$$\begin{aligned} g'(x) &= e^{x \cdot \ln(2)} \cdot \ln(2) \\ &= e^{\ln(2^x)} \cdot \ln(2) \\ &= 2^x \cdot \ln(2) \end{aligned}$$

The derivative of an exponential is ITSELF, Times the \ln of the base

In general, given an exponential function $f(x) = b^x$, $b > 0$, then

$$f'(x) = b^x \cdot \ln(b) \quad \text{— accounts for the non-base } e$$

Consider the composite function $f(x) = b^{g(x)}$.

$$f'(x) = b^{g(x)} \cdot \ln(b) \cdot g'(x) \quad \text{Chain Rule}$$

Example 5.2.2

From your text: Pg 240 #1. Differentiate:

$$\begin{aligned} \text{a) } y &= 2^{3x} \\ y' &= 2^{3x} \cdot \ln(2) \cdot 3 \\ &= 2^{3x} \cdot 3 \ln(2) \end{aligned}$$

$$\begin{aligned} \text{d) } w &= 10^{(5-6n+n^2)} \\ \frac{dw}{dn} &= 10^{(5-6n+n^2)} \cdot \ln(10) \cdot (2n-6) \end{aligned}$$

Example 5.2.3

From your text: Pg. 240 #2b) Differentiate

$$\begin{aligned} y &= x \cdot 3^{x^2} \\ y' &= (1) \cdot 3^{x^2} + x \cdot (3^{x^2} \cdot \ln(3) \cdot (2x)) \\ &= 3^{x^2} (1 + 2x^2 \cdot \ln(3)) \end{aligned}$$

Class/Homework for Section 5.2

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