# 5.3 Optimization with Exponential Functions

For this section we will utilize what we know about finding critical values (setting a derivative to zero), and testing those critical values for max/min (by plugging the c.v. into the second derivative). We'll examine two problems from your text.

**Example 5.3.1**

From your text: Pg. 245 #3

The squirrel population in a small self-contained forest was studied by a biologist. The biologist found that the squirrel population, $P$, measured in hundreds, is a function of time, $t$, where $t$ is measured in weeks. The function is $P(t) = \frac{20}{1 + 3e^{-0.02t}}$. $\Rightarrow P(t) = \frac{20}{1+3e^{-0.02t}}$

a. Determine the population at the start of the study, when $t = 0$.

b. The largest population the forest can sustain is represented mathematically by the limit as $t \to \infty$. Determine this limit.

c. Determine the point of inflection.

d. Graph the function.

e. Explain the meaning of the point of inflection in terms of squirrel population growth.

a) $P(0) = \frac{20}{1+3e^0} = 5 \Rightarrow 500$ squirrels.

b) $\lim_{t\to\infty} \left(P(t)\right) = \lim_{t\to\infty}\left(\frac{20}{1+3e^{-0.02t}}\right) = 20 \Rightarrow 2000$ squirrels is the most the forest can sustain

c) $P(t) = 20\left(1+3e^{-0.02t}\right)^{-1}$

$$P'(t) = -20\left(1+3e^{-0.02t}\right)^{-2}\left(3e^{-0.02t}\cdot(-0.02)\right)$$

$$= \frac{(1.2)e^{-0.02t}}{\left(1+3e^{-0.02t}\right)^2}$$

Note: $P'(t) > 0 \;\; \forall\, t \in D_P$

$\Rightarrow P(t)$ is always increasing.

Further $P'(t) \neq 0$

$\Rightarrow$ no critical values

$$P''(t) = \frac{(1.2)e^{-0.02t}(-0.02)\left(1+3e^{-0.02t}\right)^2 - (1.2)e^{-0.02t}\left(2\left(1+3e^{-0.02t}\right)\left(3e^{-0.02t}\cdot(-0.02)\right)\right)}{\left(1+3e^{-0.02t}\right)^4}$$

$$= \frac{(1.2)e^{-0.02t}(-0.02)\cancel{\left(1+3e^{-0.02t}\right)}\left(\left(1+3e^{-0.02t}\right) - 2\left(3e^{-0.02t}\right)\right)}{\left(1+3e^{-0.02t}\right)^{\cancel{4}\,3}}$$

$$= \frac{-0.024e^{-0.02t}\left(1-3e^{-0.02t}\right)}{\left(1+3e^{-0.02t}\right)^3}$$

set to zero for P.P.O.I

$$\Rightarrow -0.024e^{-0.02t}\left(1-3e^{-0.02t}\right) = 0$$

$$\Rightarrow 1-3e^{-0.02t} = 0$$

$$\Rightarrow e^{-0.02t} = \frac{1}{3}$$

ln both sides

$$\Rightarrow -0.02t = \ln\left(\tfrac{1}{3}\right)$$

$$\Rightarrow t = \frac{\ln\left(\frac{1}{3}\right)}{-0.02}$$

$$\doteq 55 \text{ wks}$$

"Test" $t = 55$ w/ $2^{ND}$ derivative to see if concavity changes

54    55    56

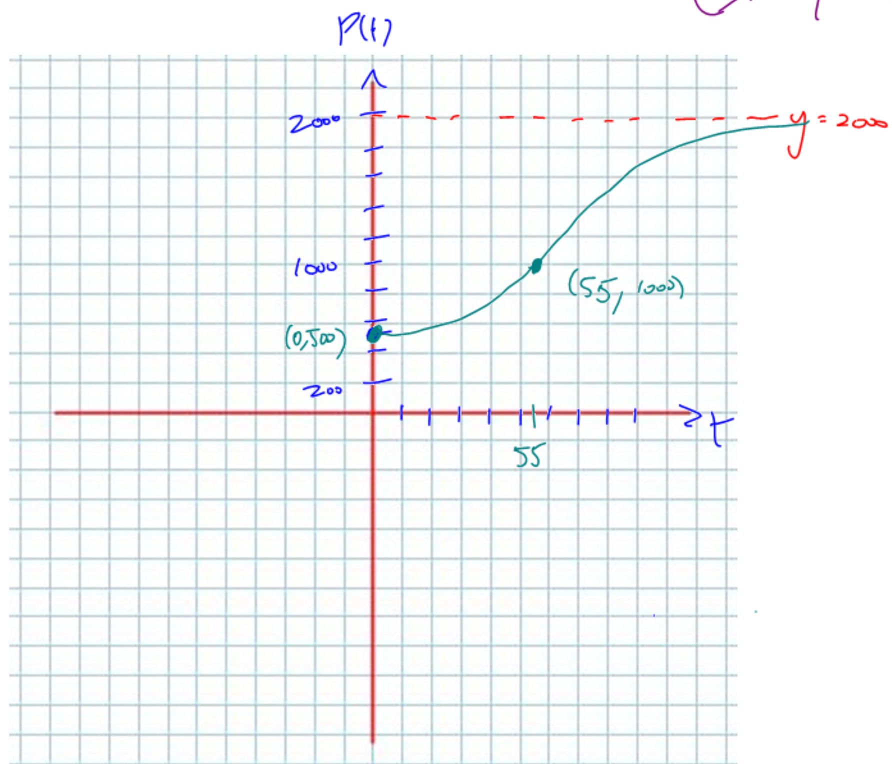
$P''(54) > 0$     $P''(56) < 0$

ccu     ccd

$\therefore t = 55$ wks is a POI.

POI : $(55, P(55))$

$= (55, 10)$ 1000 squirrels




e) The P.O.I represents the time when the squirrel's population growth begins to slow.

**Example 5.3.2**

From your text: Pg. 247 #12b

Determine the max and min values for the function $y = x \cdot e^{x} + 3$ (don't graph)

we will need find values

$$y' = (1)e^{x} + xe^{x}$$

$$= e^{x}(1+x) \quad \text{set to zero}$$

can't be zero

$$\Rightarrow e^{x}(1+x) = 0$$

$$\Rightarrow x = -1$$

Test the c.v. $x=-1$ to see if it is WHERE a max or min occurs. 2 Tests

① 1st derivative test

$-2 \quad -1 \quad 0$

$y'|_{x=-2} < 0 \qquad y'|_{x=0} > 0$

Local min

② 2nd derivative test

$$y' = e^{x}(1+x)$$

$$\Rightarrow y'' = e^{x}(1+x) + e^{x}(1)$$

$$= e^{x}(2+x)$$

Test $y''|_{x=-1} = e^{-1}(2+(-1)) > 0$ ☺ ccu

$\Rightarrow x=-1$ is at a local min

**Class/Homework for Section 5.3**

*Pg. 245 – 247 #4, 6, 8, 12cd, 13*

$\therefore$ The min value is $y|_{x=-1} = (-1)e^{-1} + 3 = \frac{-1}{e} + 3 = \frac{3e-1}{e} \doteq 2.6$