Hwk Check

8. A colony of bacteria in a culture grows at a rate given by $N(t) = 2^{\frac{t}{5}}$, where N is the number of bacteria t minutes from the beginning. The colony is allowed to grow for 60 min, at which time a drug is introduced to kill the bacteria. The number of bacteria killed is given by $K(t) = e^{\frac{t}{3}}$, where K bacteria are killed at time t minutes.

   a. Determine the maximum number of bacteria present and the time at which this occurs.

   b. Determine the time at which the bacteria colony is obliterated.

$$P(t) = \begin{cases} N(t), & 0 \le t \le 60 \\ N(t) - K(t-60), & t > 60 \end{cases} = \begin{cases} 2^{t/5}, & 0 \le t \le 60 \\ 2^{t/5} - e^{\frac{t-60}{3}}, & t > 60 \end{cases}$$

$0 \le t \le 60$ $\quad P'(t) = 2^{t/5} \cdot \ln(2) \cdot \left(\frac{1}{5}\right)$ set to zero $\quad \frac{t}{5} = \frac{1}{5}t$

$\Rightarrow 2^{t/5} \cdot \ln(2) \cdot \left(\frac{1}{5}\right) = 0$ impossible $\Rightarrow$ no C.V.

$\Rightarrow$ max for $0 \le t \le 60$ is $N(60) = 2^{12}$
$= 4096$

$t > 60$ $\quad p'(t) = 2^{t/5} \cdot \ln(2) \cdot \left(\frac{1}{5}\right) - e^{\frac{t-60}{3}}\left(\frac{1}{3}\right)$ set to zero

$$\Rightarrow \frac{2^{t/5} \cdot \ln(2)}{5} - \frac{e^{\frac{t-60}{3}}}{3} = 0 \quad \times 15$$

$$\Rightarrow 2^{t/5} \cdot 3\ln(2) - 5e^{\frac{t-60}{3}} = 0$$

$t = 98.2$ min (graphing calc)

$\Rightarrow$ 38.2 minutes after the drug intro

b) $P(t) = 0$. This will happen after $t > 60$

$$\Rightarrow \quad 2^{t/5} - e^{\frac{t-60}{3}} = 0$$

$$\Rightarrow \quad 2^{t/5} = e^{\frac{t-60}{3}}$$ (ln both sides)

$$\Rightarrow \quad \ln\left(2^{t/5}\right) = \ln\left(e^{\frac{t-60}{3}}\right)$$

$$\Rightarrow \quad t/5 \cdot \ln(2) = \frac{t-60}{3}$$

$$\Rightarrow \quad t\left(\frac{\ln(2)}{5}\right) - \frac{t}{3} = -20$$

$$\Rightarrow \quad t\left(\frac{\ln(2)}{5} - \frac{1}{3}\right) = -20$$

$$\Rightarrow \quad t = \frac{-20}{\frac{\ln(2)}{5} - \frac{1}{3}} \times \frac{-15}{-15}$$

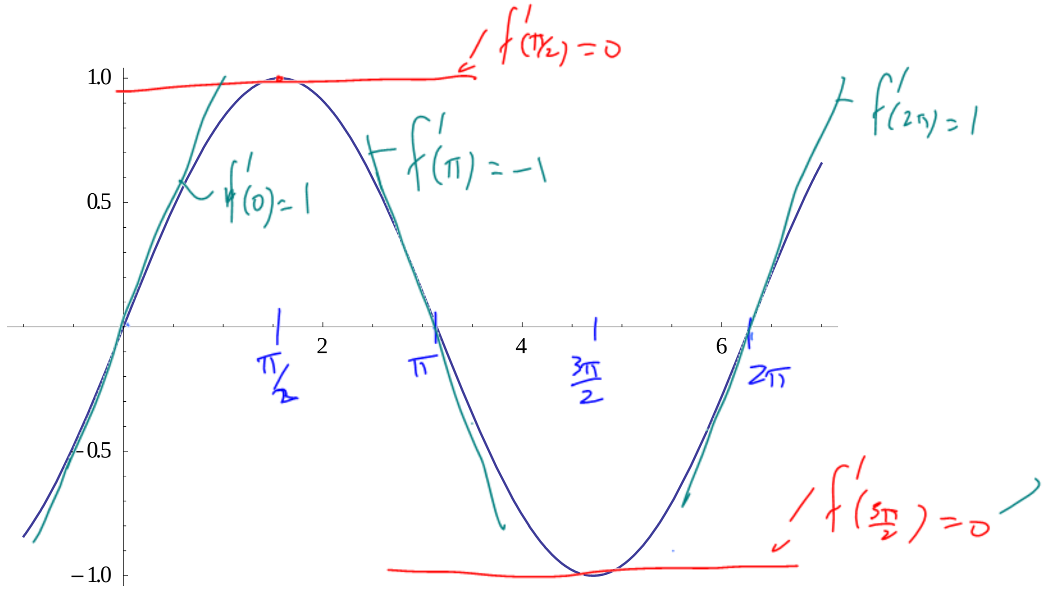
$$= \frac{300}{5 - 3\ln(2)} = 102.7 \text{ minutes}$$

$\Rightarrow$ 42.7 minutes after drug is introduced.

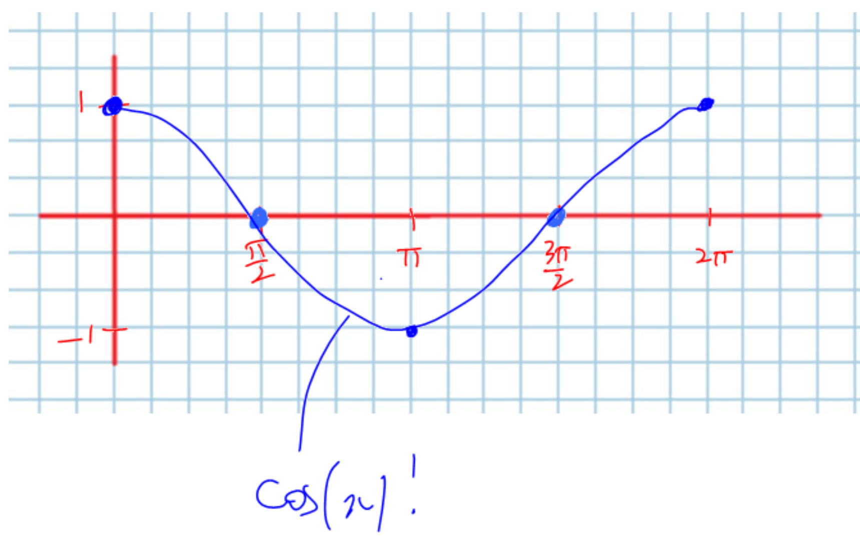
5.4 The Derivatives of Sine and Cosine

We begin with a "geometric analysis" of the function $f(x) = \sin(x)$, considering its derivative (geometrically) at various domain values.

$$f(x) = \sin(x)$$



$$f'(x) = ?$$



Definition 5.4.1

Given $f(x)=\sin(x)$

Then $\dfrac{df}{dx}=f'(x)=$ $\cos(x)$

$\tan(x)=\dfrac{\sin(x)}{\cos(x)}$

Given $g(x)=\cos(x)$

prove $\dfrac{d}{dx}(\tan(x))=\sec^2 x$

Then $\dfrac{dg}{dx}=g'(x)=$ $-\sin(x)$

Example 5.4.1

Determine the derivative of

a) $y=\sin(3x^2-5x)$

$y'=\cos(3x^2-5x)\cdot(6x-5)$

$=(6x-5)\cdot\cos(3x^2-5x)$

b) $f(x)=e^{\sin(x)+\cos(x)}$

$f'(x)=e^{\sin(x)+\cos(x)}\cdot(\cos(x)-\sin(x))$

c) $g(x)=\sin^3(4x)$

$g(x)=(\sin(4x))^3$

$g'(x)=3(\sin(4x))^2\cdot(\cos(4x)\cdot 4)$

$=12\sin^2(4x)\cdot\cos(4x)$

d) $y=\dfrac{\cos(4x)}{e^{\sin(x)}}$

$y'=\dfrac{-4\sin(4x)\cdot e^{\sin(x)}-\cos(4x)\cdot e^{\sin(x)}\cdot\cos(x)}{(e^{\sin(x)})^2}$

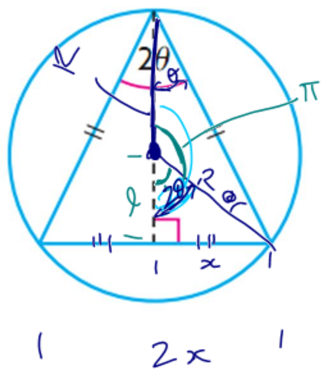
$=\dfrac{\cancel{e^{\sin(x)}}(-4\sin(4x)-\cos(4x)\cdot\cos(x))}{(e^{\sin(x)})^{\cancel{2}}}$

$=\dfrac{-4\sin(4x)-\cos(4x)\cdot\cos(x)}{e^{\sin(x)}}$

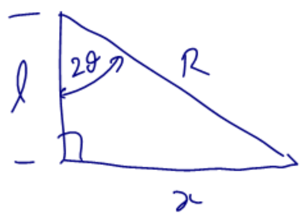
Example 5.4.2

From your text: Pg. 257 #13

An isosceles triangle is inscribed in a circle of radius **R**. Find the value of θ that maximizes the area of the triangle.



$A = \frac{1}{2} b \times h$



$\sin(2\theta) = \dfrac{x}{R}$

$\Rightarrow x = R\sin 2\theta$

$\Rightarrow b = 2R\sin(2\theta)$

$\cos(2\theta) = \dfrac{\ell}{R}$

$\Rightarrow \ell = R\cos(2\theta)$

$\Rightarrow h = R\cos\theta + R$

$b = 2x$

$h = \ell + R$

$$\therefore A(\theta) = \frac{1}{2}\Big(2R\sin(2\theta)\Big)\Big(R\cos(2\theta) + R\Big)$$ factor out R

$$= R^2\Big(\sin(2\theta)\Big)\Big(\cos(2\theta) + 1\Big)$$

$$\Rightarrow A'(\theta) = R^2\Big[2\cos(2\theta)\big(\cos(2\theta) + 1\big) + \big(\sin(2\theta)\big)\big(-2\sin(2\theta)\big)\Big]$$

set to zero and $\div\ 2R^2$

Class/Homework for Section 5.4

Pg. 256 – 257 #1ace, 2bcde, 3bcf, 5, 6ac, 7, 9, 12

$$\Rightarrow \cos^2(2\theta) + \cos(2\theta) - \sin^2(2\theta) = 0$$

$$\Rightarrow \cos^2(2\theta) + \cos(2\theta) - \left(1 - \cos^2(2\theta)\right) = 0$$

$$2\cos^2(2\theta) + \cos(2\theta) - 1 = 0$$

quadratic in $\cos(2\theta)$

exactly like

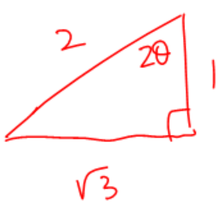
$$2x^2 + x - 1 = 0$$

$$(2x - 1)(x + 1) = 0$$

$$x = \tfrac{1}{2} \text{ or } x = -1$$

$$\Rightarrow \left(2\cos(2\theta) - 1\right)\left(\cos(2\theta) + 1\right) = 0$$

$$\Rightarrow \cos(2\theta) = \tfrac{1}{2} \quad \text{or} \quad \cos(2\theta) = -1$$



$$2\theta = \frac{\pi}{3}$$

$$\Rightarrow \theta = \frac{\pi}{6}$$

$$\Rightarrow 2\theta = \pi$$

impossible

Note: we actually do need to test if $\theta = \frac{\pi}{6}$ gives a max area $\Rightarrow$ 2^{ND} deriv. or 1^{ST} deriv test } do on your own