# 5.6 The Derivatives of Logarithms

$\ln(x) = \log_e(x)$

We will consider two "types" of logarithms: The Natural Logarithm (with base *e*), and The General Logarithm (with base *b*). We'll begin with…

## The Derivative of The Natural Logarithm

$y = \log_b(x) \iff b^y = x$

Given $y = \ln(x)$, determine $\dfrac{dy}{dx} = y'$

**Note**: It's always a good idea to **work with things you already know about**. For example we know a lot about the derivative of the **natural exponential function**!

We know something about the inverse of $\ln(x)$!

Given $y = \ln(x)$, invert to an exponential equation

$\Rightarrow e^y = x \quad (*)$

Take the derivative of both sides w.r.t. $x$

$$\frac{d}{dx}\left(e^y\right) = \frac{d}{dx}(x)$$

$$e^y \cdot y' = 1$$

$$\Rightarrow y' = \frac{1}{e^y}$$

but we want $y'$ as a fn of $x$

BUT $e^y = x$ by $(*)$

$$\Rightarrow y' = \frac{1}{x}$$

**The Chain Rule:**

Given $f(x) = \ln\left(g(x)\right)$, then

composite fn

$$f'(x) = \frac{1}{g(x)} \cdot g'(x) = \frac{g'(x)}{g(x)}$$

**Example 5.6.1**

a) Differentiate $y = \ln(\sin(x))$

$$y' = \frac{\cos(x)}{\sin(x)} = \cot(x)$$

b) Differentiate $f(x) = (\ln(x))^3$

$$f'(x) = 3(\ln(x))^2 \cdot \frac{1}{x}$$

c) Differentiate $y = \ln(x^3)$

2 METHODS: simplify first

$$y' = \frac{1}{x^3} \cdot 3x^2 = \frac{3}{x}$$

$$y = 3 \cdot \ln(x)$$

$$y' = 3 \cdot \frac{1}{x} = \frac{3}{x}$$

## The Derivative of The General Logarithm

Given $y = \log_b(x)$, determine $\frac{dy}{dx}$.

Invert

$$b^y = x$$

differentiate both sides wrt $x$

$$\frac{d}{dx}(b^y) = \frac{d}{dx}(x)$$

$$b^y \cdot \ln(b) \cdot y' = 1$$

$$y' = \frac{1}{b^y \cdot \ln(b)} = \frac{1}{x \cdot \ln(b)}$$

**Example 5.6.2**

Differentiate $g(t)=\log_5\left(3t^2\right)$

$$g'(t)=\frac{1}{3t^2\cdot\ln(5)}\cdot 6t$$

$$=\frac{6t}{3t^2\cdot\ln(5)}$$

$$=\frac{2}{t\cdot\ln(5)}$$

*Class/Homework for Section 5.6*

*Pg. 575 #3abc, 4def, 5, 6, 10*